

# Analysis of Statically Indeterminate Beams Using an Innovative Approximate Method

Dipak J.Varia

Research scholar, Rai University, Ahmedabad,  
Assistant Professor Applied Mechanics Department,  
Government Engineering College,  
Patan, Gujarat, India.  
djvaria@gmail.com

Dr.Harshvadan S.Patel

Professor Applied Mechanics Department,  
Government Engineering College,  
Patan, Gujarat, India.  
dr.hspatel@yahoo.com

**Abstract**— There are numerous classical and matrix methods are available for analysis of Indeterminate beams. In recent times high-rise buildings construction are increasing in big cities. Analysis of such structures with complex geometry and different combinations of loads are tedious. This paper is an attempt to present novel method of analysis for a indeterminate beam using an approximate approach. This paper also intends to compare the results obtained by approximate method of structural analysis to the values obtained from other conventional methods. The results obtained by the approximate approach are found comparable with those found out by established conventional methods.

**Keywords:** Structure analysis, Continuous Beam, Approximate analysis, Indeterminate Beam

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## I. INTRODUCTION

Kanat Burak Bozdogan and Duygu Ozturk[1] have presented an approximate method based on the continuum approach and transfer matrix method for free vibration analysis of multi bay coupled shear wall. They conclude that the method is simple and accurate. An approximate hand method for estimating horizontal loading is presented by J.C.D.Hoenderkamp and H.H. Snijder[2]. They conclude that the information obtained from this method should give the design engineer an easy means of comparing the suitability of alternative structural proposals, in addition to providing initial structural data for a more accurate analysis, or allowing a check on the reasonableness of the final output of a computer analysis. R.A.Behr, C.H.Goodspeed, R.M.Henry[3] have present the note to alert structural engineers to the potential errors in textbook methods of approximate structural analysis. A reliable, reasonably accurate approximate method of structural analysis for symmetric, rectangular frames under symmetric vertical loading has been developed by R.A.Behr, E.J.Grotton and C.A.Dwinal[4]. Okonkwo V.O, Aginam C.H. and Chidolue C.A[5] developed the mathematical model for evaluation of the internal support moments of a uniformly loaded continuous beam of equal span and the number of spans, taking the uniformly distributed load on the beam to be equal for all spans. An overview of various approximate method was briefly done by Life John and Dr. M.G. Rajendran[6]. This paper also intends to compare revised method of structural analysis to the values obtained from STAAD.pro. Design charts are developed for selection of beam and reinforcement when the beam moment is available by S.N.Khuda and Anwar[7].

In recent times High-rise building construction are increasing in big cities. Analysis of such structures with complex geometry and different combinations of loads are tedious. Analysis of indeterminate beam like continuous beam can be obtained by methods like Moment distribution, Slope deflection, Energy methods etc[8-10]. In all the above mentioned methods one has to carry out iterations or to solve equations and that is most time and computer memory

consuming part. To overcome these difficulties authors have evolved new approximate analysis method for indeterminate beams.

## II. BASIC TERMS

The method is dependent on four basic terms which are explained as under:

### (a) Corrected Member Stiffness (K)

The Corrected Member stiffness of a beam is the product of fixity coefficient and relative flexural stiffness (EI/L).

$$K = C_f \times (EI/L) \quad \text{..... (1)}$$

Where  $C_f$  = Fixity Coefficient.

### (b) Relative Deformation Coefficient ( $C_r$ )

Relative deformation coefficient is deformation of far end of a beam member if unit deformation is applied at the near end of the member. If unit rotation is applied to the near end of a fixed beam, then  $C_r$  at the far end is 0 due to fixed support. But in case of propped cantilever, if unit rotation is applied to a fixed near end, then  $C_r$  at far end is 0.5 due to hinged support. In multi-span beams, the value of  $C_r$  in case of intermediate supports is computed using the following relation:

$$C_r = K_1 / [2(K_1 + K_2)] \quad \text{..... (2)}$$

Where  $K_1$  and  $K_2$  are corrected member stiffness of members meeting at a joint.

### (c) Fixity Coefficient ( $C_f$ )

Fixity coefficient gives the fixity provided by far end. The value of  $C_f$  at near end is always 1 and the same for far end is dependent on  $C_r$  at far end.

$$C_f = 1 - C_r / 2 \quad \text{..... (3)}$$

**(d) Actual Deformation (A<sub>D</sub>)**

Actual deformation of a joint is joint deformation due to unit deformation of any other joint. It is given as the product of A<sub>D</sub> of preceding joint and the C<sub>r</sub> of the joint under consideration.

$$A_D(i) = A_D(i-1) \times C_r(i) \dots\dots\dots (4)$$

Where, (i) = Joint Index.

**III. STEPS FOR CONTINUOUS BEAM**

- (1) Choose suitable sign conventions for forces and deformations.
- (2) Identify any one unknown action at a joint.
- (3) Compute relative deformation co-efficient C<sub>r</sub> and fixity co-efficient C<sub>f</sub> for all joints except at joint where unknown action is identified. Start computing from extreme support/s and move towards joint where unknown action is identified.
- (4) Take A<sub>D</sub> equal to unity i.e. deformation corresponding to unknown force. Start computing actual deformation A<sub>D</sub> at each joint from joint where unit deformation is applied.
- (5) Compute fixed end actions corresponding to deformations.
- (6) Compute the sum of multiplication of actual deformations A<sub>D</sub> with fixed end actions. The sum yields value of identified unknown action.

**IV. APPLICATION**

To illustrate the application of innovative approximate approach problem of three span continuous beam as per Fig. 1 is selected. For obtaining moment at different joints one should calculate C<sub>r</sub> and C<sub>f</sub> for that joints. Once the C<sub>r</sub> has been calculated it is very easy to obtain the moment at required joint. In this study authors propose approximate values for C<sub>r</sub> and C<sub>f</sub> and overcome the difficulty of calculation of C<sub>r</sub> and C<sub>f</sub>. One can take the approximate values of C<sub>r</sub> and C<sub>f</sub> as per Table 1. Sample calculations are shown for joints in Table 2-5. Finally comparison with exact values and % error are presented in Table 6.

Table 1. Approximate value for C<sub>r</sub> and C<sub>f</sub>

Type of Joint	C <sub>r</sub>	C <sub>f</sub>
Extreme (Fixed)	0	1
Not extreme (Internal)	0.25	0.875

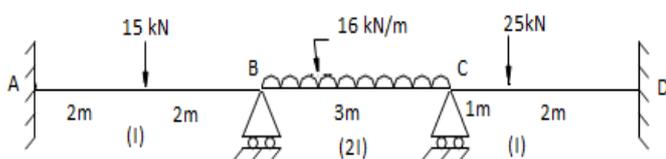


Figure 1. Three span continuous beam problem

Table 2. Computation for support moment at A

Joint	A	B	C	D
C <sub>r</sub>	-	0.25	0.25	0
C <sub>f</sub>	-	0.875	0.875	1
A <sub>D</sub>	1	-0.25	0.0625	0

FEM	7.5	4.5	-0.89	-5.56
A <sub>D</sub> * FEM	7.5	-1.125	-0.056	0

M<sub>A</sub> = ∑ A<sub>D</sub>\* FEM = 6.319 kN.m  
 Ref. value of M<sub>A</sub> = 6.76 kN.m  
 Percentage Error = -6.52%

Table 3. Computation for support moment at B

Joint	A	B	C	D
C <sub>r</sub>	0	-	-	0.25
C <sub>f</sub>	1	-	-	0.875
A <sub>D</sub>	0	0.7	0.3	-0.075
FEM	-7.5	7.5	12	-0.89
A <sub>D</sub> * FEM	0	5.25	3.6	0.067

$$AD_{BA} = 1 - \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= 1 - \frac{C_{fA}(I/L)_{BA}}{C_{fA}(I/L)_{BA} + C_{fC}(I/L)_{BC}}$$

$$= 1 - \frac{1 * (I/4)}{1 * (I/4) + 0.875(2I/3)}$$

$$= 0.7$$

$$AD_{BC} = 1 - 0.7 = 0.3$$

M<sub>B</sub> = ∑ A<sub>D</sub>\* FEM = 8.917 kN.m  
 Ref. value of M<sub>B</sub> = 8.98 kN.m  
 Percentage Error = -0.70%

Table 4. Computation for support moment at C

Joint	A	B	C	D
C <sub>r</sub>	0	0.25	-	-
C <sub>f</sub>	1	0.875	-	-
A <sub>D</sub>	0	0.0909	0.3636	0.6364
FEM	-7.5	4.5	12	11.11
A <sub>D</sub> * FEM	0	0.4090	4.3632	7.0704

M<sub>C</sub> = ∑ A<sub>D</sub>\* FEM = 11.8426 kN.m  
 Ref. value of M<sub>C</sub> = 12.06 kN.m  
 Percentage Error = - 1.8026%

Table 5. Computation for support moment at D

Joint	A	B	C	D
C <sub>r</sub>	0	0.25	0.25	-
C <sub>f</sub>	1	0.875	0.875	-

$A_D$	0	0.0625	-0.25	1
FEM	-7.5	-4.5	0.89	5.56
$A_D * FEM$	0	-0.2812	-0.02225	5.56

$M_D = \sum A_D * FEM = 5.2565 \text{ kN.m}$

Ref. value of  $M_D = 5.07 \text{ kN.m}$

Percentage Error = 3.6785%

### V. SUMMARY

Table 6.% error for computed moment

Joint	By Approximate approach	By conventional method	% error
Moment at A	6.319	6.76	-6.52
Moment at B	8.917	8.98	-0.70
Moment at C	11.8426	12.06	-1.8026
Moment at D	5.2565	5.07	3.6785

### CONCLUSIONS

- This paper provides an overview to the innovative approximate method.
- The solution of three span continuous beam has been demonstrated by problem.
- The results obtained are quite close to the standard values.
- One can analyze the continuous beam with 93% accuracy.

- The same method can be extended to solve a continuous beam with more number of spans.
- From this it can concluded that the method evolved is innovative, simple and acceptable.

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