

Harmonic Analysis of Square Plate with and Without Uncertain Parameters

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Abstract— Forced vibration analysis of plate is growing research subject as plates are used for manufacturing many mechanical structures. The analytical solutions have been found for plates with specified mass and stiffness but for the plates with uncertain mass and stiffness are studied very limitedly. The uncertain parameters play an important role in the design of practical mechanical system. It becomes important to study its effects on mechanical system for different frequency domain. Here in this paper structural element square plate with all edge fixed boundary condition is selected on which mass, stiffness and combination of mass and stiffness uncertainty are considered. By using Finite Element Method technique finding out, how plate is dynamically behaves in forced vibration. In the harmonic analysis the response of bare plate is compared with the plate of mass, stiffness and mass-stiffness combination uncertainty. During comparison it is found that uncertain parameters have adverse effects on the dynamic behavior of the plate. The complete plate characteristics were changed.

Keywords- *Vibration Analysis, Harmonic Analysis, Forced Vibration, Uncertain Parameters, FEM and Ansys software.*

I. INTRODUCTION

The machine structures, aircraft, household goods and civil engineering structure are mostly made up of plates. The study of mathematical models of plates, involve physical and geometric parameters such as mass density ρ , elastic modulus E , Poisson's ratio ν , lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes, structural loading and environmental conditions [1]. These uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered in most of the engineering design. This concept of uncertainty plays an important role in investigation of various engineering problems [2]. The dynamic behaviour of each element is derived and the solution of the structure as a whole is reconstituted by attaching each element together at its node points. Predicting the dynamic response of a vibrating system generally involves determining the equations of motion of the structure and solving them for given boundary conditions [3]. Finite element methods are one of the most widely used deterministic techniques. For this technique, a structure is divided into a number of elements. These methods are extensively used to predict the linear dynamic response of structures in the low frequency region. For more complex systems, the equations of motion can be approximated using various deterministic modeling techniques such as finite element analysis (FEA) [4] and dynamic stiffness techniques. In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of given set of design parameters [5] Energy methods such as Statistical Energy

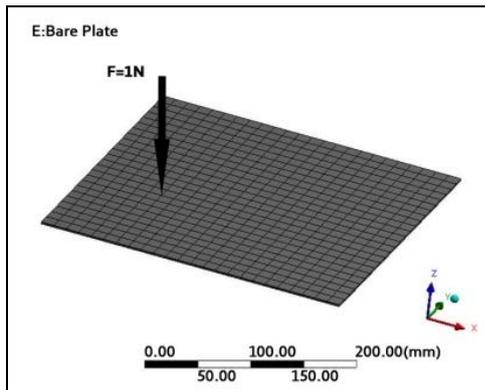
Analysis (SEA) are appropriate dynamic predictive techniques at high frequencies. SEA is a technique which models a structure in terms of its gross system properties. In this method, a complete system is described in terms of energy flow between various subsystems [6, 7]. The natural frequencies and mode shapes can then be used to predict the response due to an applied excitation [8].

Here in this paper square plate is used with all edge fixed boundary condition. In this square plate two major uncertain parameters are consider i.e. mass and stiffness. Hence the analysis is done on bare plate and plate with uncertainty due to mass, stiffness and combination of mass and stiffness. For all these combination harmonic analysis is done and the response of plate for all these parameters were found using FEM.

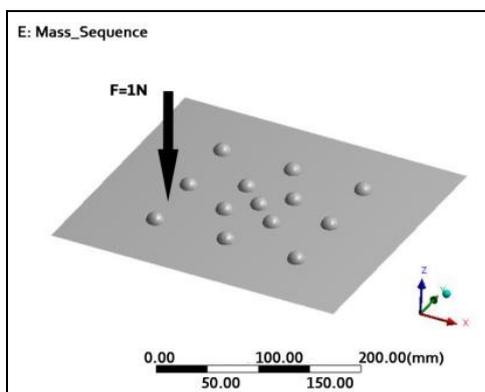
II. MODELING OF SQUARE PLATE WITH AND WITHOUT UNCERTAINTY

A square plate of dimension $304.8 \times 304.8 \times 2$ mm of steel material with properties of $\rho = 7.86 \times 10^{-9}$ tonne/mm³, $\nu = 0.3$, $Y = 2e^5$ MPa was created. Applied all material properties and all edge fixed boundary condition. The element type selected is quadratic shell element 181. SHELL181 is suitable for analyzing thin to moderately-thick shell structures. The spring stiffness is taken 2452.5×10^{-3} N/mm. The stiffness element used is 'COMBIN14' [10]. Total mass of plate is 1.5×10^{-3} tonne. Mass uncertainty is taken 2% of total mass [9]. Number of masses taken is 13. The 'Mass21 element' type is taken for these uncertain masses. They are sequentially placed on a plate.

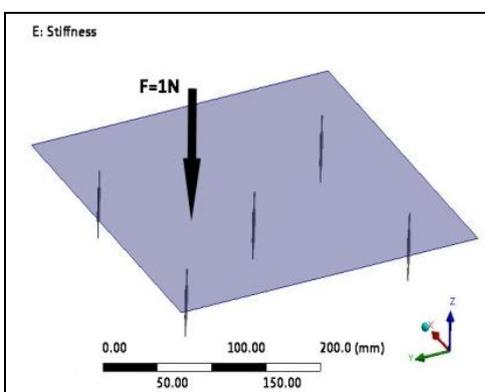
In this force vibration study, force of 1 N is applied at the node location (75 mm, 152.4 mm) of the all plates. The frequency range is given from 0 to 3000 Hz. In this study of damped force vibration as per general rule of steel material, constant damping ratio of 1% or 0.01 is taken. The modeling of plate for bare as well as for the plate with all uncertain parameters is shown in Figure 1. Harmonic analysis of plate with and without uncertainty is solved in ANSYS 14.5, FEA software using the full method. The full method uses the full system matrices to calculate the harmonic response (no matrix reduction). The matrices may be symmetric or unsymmetrical. It uses full matrices, so no mass matrix approximation is involved [10].



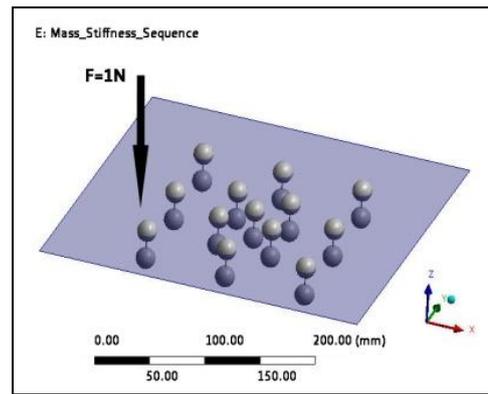
(a)



(b)



(c)



(d)

Figure 1. Modeling of plate for harmonic analysis: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty and (d) mass and stiffness uncertainty.

III. EFFECT OF UNCERTAIN PARAMETERS ON THE VIBRATIONAL CHARACTERISTICS OF PLATE WITH ALL EDGE FIXED CONDITION USING HARMONIC ANALYSIS

During a forced vibration analysis, it is found that the fundamental mode for the bare plate was excited at the frequency 189.06 Hz, where the highest value of amplitude $1.8E-0.2\text{mm}$ is obtained. All the frequency of plate with uncertain parameters is compared to the bare plate. The plate with mass uncertainty is excited at 184.77 Hz as compared with the bare plate frequency and there is a variation of 3.17%. Due to the addition of stiffness uncertainty, the plate is excited at 207.13 Hz and the variation in frequency value is 9.96% as compared with the bare plate. As the plate with mass and stiffness uncertainty excited at 203.64 Hz and the variation in frequency is 7.71%. The variation in the frequency of bare plate and plate with all the uncertain parameters are shown in the bar chart in Figure 2.

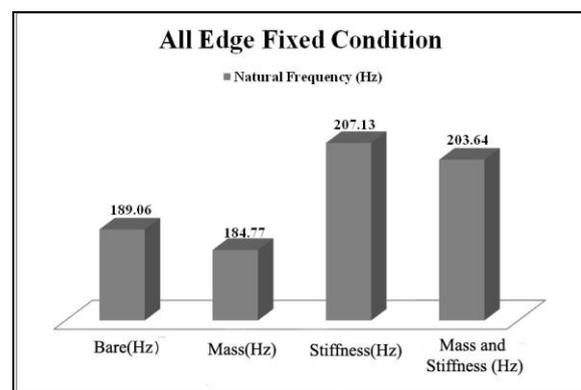


Figure 2. Bar Chart showing variation in fundamental mode frequency for all edge fixed condition plate.

IV. FRF PLOT OF THE PLATE FOR ALL EDGE FIXED CONDITIONS ON LINEAR SCALE

The all edge fixed condition plate is excited by external force and the response of plate is plotted on the FRF graph as shown in Figure 3. The first fundamental mode is excited and single pick obtained in the graph. The frequency of the bare plate (green colour) at this mode is 189.06 Hz and the amplitude 0.0183450mm. The mass (black colour) uncertainty is added due to this the frequency obtained 184.77 Hz and amplitude 0.01835mm. When the stiffness (violet colour) uncertainty is added, a very prominent pick with a major shift in frequency of 207.13Hz and large amplitude of 1.83670 mm obtained. Due to mass and stiffness uncertainty (orange colour), major frequency shift of 203.64Hz and amplitude of 0.014809 mm is obtained. All the other parameters are not observed in the graph because of linear scale only stiffness parameter is prominently obtained.

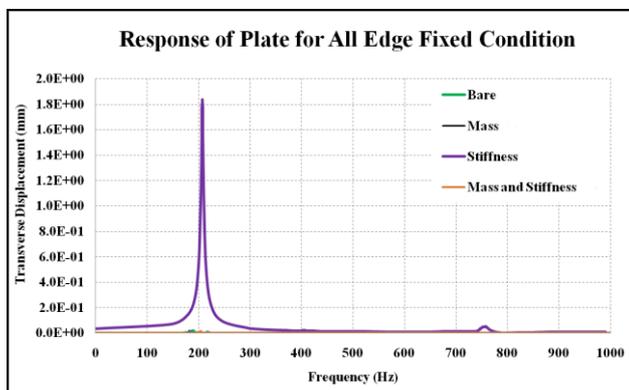


Figure 3. Graph showing the response of plate for harmonic excitation of all edge fixed condition.

V. FRF PLOT OF THE PLATE FOR ALL EDGE FIXED CONDITIONS ON LOG-LOG SCALE

The FRF of plate for all edge fixed condition is shown in Figure 4. The response of the bare plate is compared with the all other uncertain parameters. In this FRF of all edge fixed condition plate, the external excitation is applied. The first fundamental mode of bare plate (green colour) is excited at 189.60 Hz frequency with the real displacement (transverse displacement) of amplitude 0.0183450mm. This is the first resonance point of bare plate. When a mass (violet colour) uncertainty is added to bare plate the frequency shifted to 183.07 Hz and small change obtained in the amplitude of 0.018293mm. Because of mass uncertainty small changes are obtained in the resonance frequency and in the amplitude. But when the stiffness (red colour) uncertainty is added the complete resonance point is shifted. The frequency at this point is 207.13Hz and large changes in amplitude is obtained at some other value as compared to bare plate. This parameter completely changes the characteristics of the plate. As and when the mass and stiffness (black colour) uncertainty is added, the resonance point is shifted to frequency 203.64Hz

and amplitude 0.014809 mm. As compared to the bare plate, response in frequency 700 Hz to 1000 Hz some different modes are excited, which were not present in the bare plate characteristics. The completely different characteristics of plate are obtained due to mass and stiffness uncertainty.

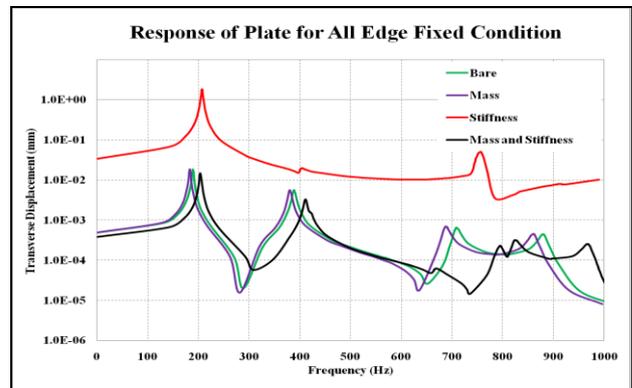


Figure 4. FRF plot on log – log scale for all edge fixed condition.

VI. CONCLUSIONS

The complete dynamic characteristic of plate is changed due to addition of uncertainties. The frequency is shifted drastically in the first fundamental mode as well as in other modes. The amplitude (transverse displacement) is changed with a very high value. Thus the complete picks of resonance are shifted and amplitude is also changing. This means that at some other value of natural frequency, the resonance point is obtained due to uncertainty, rather than which are considering previously as in bare plate. The totally different characteristic of plate is obtained at the higher modes. The plate may get failed at this higher mode. If the bare plate response is considered in the design of any plate than the plate characteristics will be completely misinterpreted. So it becomes very important to consider this uncertainty in the plate design.

VII. ACKNOWLEDGMENT

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