

## Slope Analysis with the Finite Element Method

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**Abstract-** The paper explores about the Finite Element Method (FEM) on the slope. There is a simple FEM Method having some steps to find the slope of any complex region analysis. With the help of this paper complex region is to be analyzed and calculate their failure of region of slope found in which area is to be calculated. In the paper we used principle of minimum potential energy (PMPE) method which is used to find the stresses and reaction develop in the region of slope is also done

**Keywords-** Discretization, Minimum Potential Energy Method, Global Stiffness Matrix.

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### I. INTRODUCTION

Finite Element Method (FEM) is a numerical technique to determine the field variance. It is an important technique used in CAD for modification of design of machine parts.

In this technique a complex region defines a continuum is discretized into simple geometric shapes called finite elements. The various load acting on the element along with constraints results in a set of equation.

The FEM has become a powerful tool for the numerical solution of a wide range of engineering problem. The variance from various parameters could be calculated and hence the modification required in design could be obtained. FEM is the numerical analysis technique which is used to find unknown variables. Finite elements method consist of,

- A. Finite Element Modeling (FEM)
- B. Finite Element Analysis (FEA)

### II. LITERATURE REVIEW

Generally FEM method can be done in various method such as,

- A. Principle of Minimum Potential Energy (PMPE).
- B. MESH Generation Technique.
- C. Triangular Method.

The FEM method is not only used for 1-D element, it also used for 2-D and 3-D element having multiple type of region and their shapes.

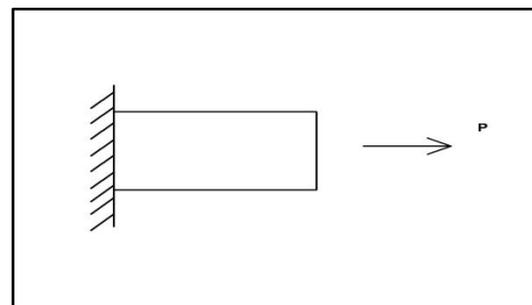


Figure 1: 1-D Element

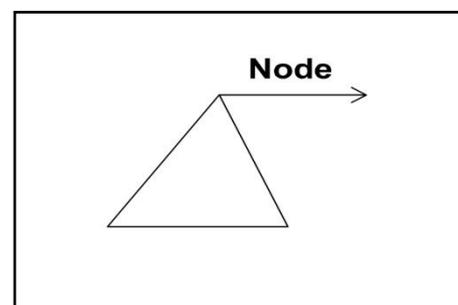


Figure 2: 2-D Element (Triangular Element)

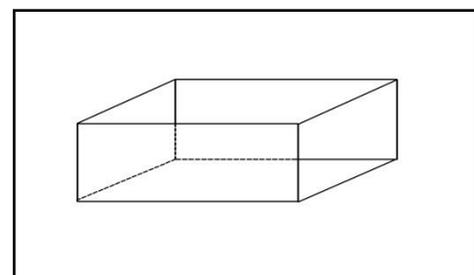


Figure 3: 3-D Element (Hexahedron Element)

### III. PRINCIPLE OF MINIMUM POTENTIAL ENERGY METHOD

Every elastic body changes its shape when force is

applied to it and region it's shape when forced is removed. When elastic body is loaded the internal energy or strain energy is depend on it. The strain energy depends on stiffness as well as displacement of the body. External forced applied on the body produced potential energy in the body.

“It states that the sum of strain energy and potential energy due to external load is Minimum if the body is in equilibrium.”

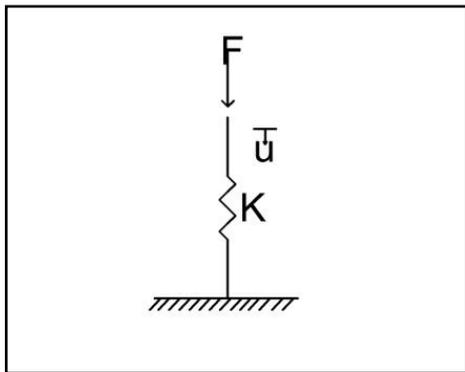


Figure 4: Stiffness element

Consider a body as shown in above figure no. 4 having stiffness “k” subjected to force “F” and having displacement “u”.

$$\text{Strain Energy, S.E.} = \frac{1}{2} Ku^2 \quad \text{-----(1)}$$

$$\text{Potential Energy, P.E.} = -F \times u \quad \text{-----(2)}$$

(“- ve” sign indicates work has to be done.)

$$\text{Total Energy, T.E.} = \text{S.E.} + \text{P.E.}$$

$$\text{Total Energy, T.E.} = \frac{1}{2} Ku^2 - F \times u \quad \text{-----(3)}$$

Hence, get minimum value of total energy.

Differentiate equation 3, with respect to “u” and equate it to zero.

$$d(\text{T.E.})/du = 0$$

$$\frac{1}{2} Ku^2 - F \times u = 0$$

$$Ku - F = 0$$

$$F = k \times u \quad \text{-----(4)}$$

When equation 4 is the equilibrium equation of component. In matrix form equation 4 can be written as,

$$\{F\} = \{k\}\{u\}$$

Where,

{F}= Modal force matrix.

{k}= Stiffness matrix.

{u}= Nodal displacement matrix.

#### IV. FORCES ON BODY OF THE SLOPE

There are generally three types of forces in the body,

##### A. Body Forces-

The force which is uniformly distributed over the entire body is called as body force.  
 e.g., Weight of the body.

##### B. Traction Force-

The force which is acting over the surface of the body is called as traction force.  
 e.g., Friction and viscous force.

##### C. Point Force-

The extending force acting at a particular point is called as point force.  
 e.g., Cantilever beam.

#### V. PROCEDURE AND DESIGN CALCULATIONS

We are finding the slope analysis of the slope. The above following figure is the demo shape image which are we need to find stresses, displacement and the reaction.

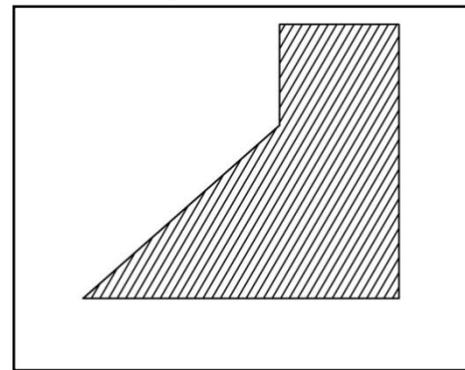


Figure 5: Demo Slope

##### A. Discretization-

First we need to discretization the slope meant that the slope is divided into number of small finite elements such as grid or mesh of these element cover the entire component.

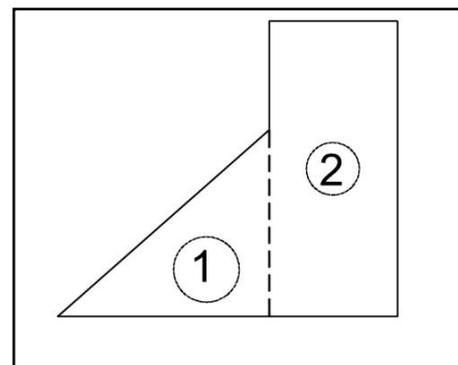


Figure 6: Demo Slope with discretization

##### B. Considering Each Element and Solve-

Equation governing to the individual elements are worked out and approximate solution to the equation of an elements find out,

a) Consider Element 1.

$$\text{Stiffness, } K_1 = \frac{A1E}{L}, \text{ N/m}$$

$$K^1 = \begin{bmatrix} k1 & -k1 \\ -k1 & k1 \end{bmatrix}$$

2) Consider element 2,

$$\text{Stiffness, } K_1 = \frac{A2E}{L}, \text{ N/m}$$

$$K^1 = \begin{bmatrix} k2 & -k2 \\ -k2 & k2 \end{bmatrix}$$

C. Global Stiffness Matrix-

Solution of individual elements together considering the connectivity of the elements.

$$K^G = K^1 + K^2$$

D. Nodal Force Matrix-

1) Displacement Matrix-

$$F = \begin{pmatrix} F1 \\ F2 \\ F3 \end{pmatrix}, \text{ N}$$

2) Nodal Displacement Matrix-

$$u = \begin{pmatrix} u1 \\ u2 \\ u3 \end{pmatrix}, \text{ mm}$$

Boundary condition are applied to the assembled equation and final solution is achieved.

Using Gauss Elimination Method, Boundary condition are applied to the assembled equation and final solution is achieved.

Use equation,

$$F = K^G u$$

E. Finding stress and Strain-

Selection is post proceeded to work out additional parameters like stresses and strain,

u= Modal displacement matrix

B= strain displacement matrix

$$B = 1/L \begin{bmatrix} -1 & 1 \end{bmatrix}$$

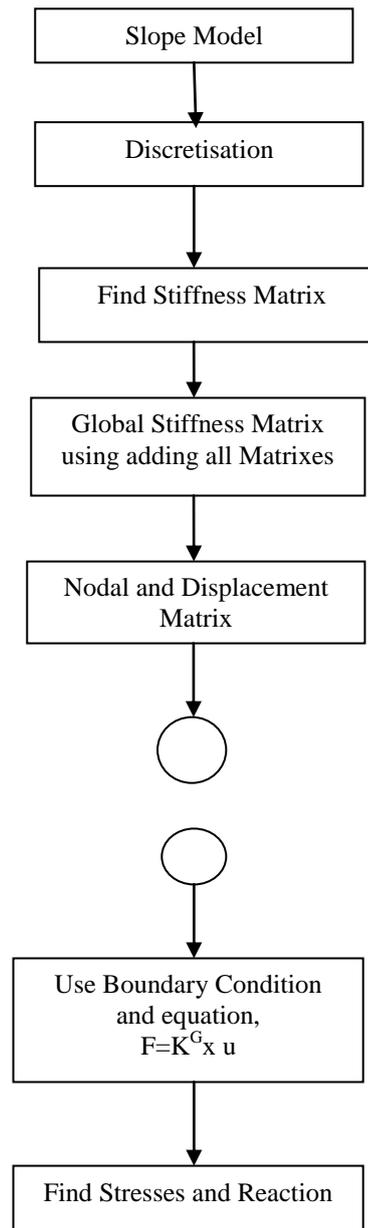
$$u = \begin{pmatrix} u1 \\ u2 \end{pmatrix}$$

F) Reaction at Supports-

$$R = [K_1 \ K_2 \ K_3], \text{ KN}$$

Results are tested for convergence using convergence test. With the help of these we can find any type of slope and also having stress and strain induced in the slope zone. The reaction supports occurs in the fixed supports of the slopes. So, the FEM is very useful method for the slope analysis.

VI. FLOWCHART TO REPRESENT STEPS-



VII. CONCLUSION

With the help of these we found the slope analysis with FEM method. This method is very useful for further calculation of complicated slope analysis with easy calculation. For further reference the FEM is the best way to solve the complicated slope and having some steps we could find the slope analysis.

Although the analysis is done in various method but these paper is based on the simple stepwise calculation using stiffness, nodal matrix and boundary conditions with the help of this type various types of complex region is to be solved.

So, slope analysis with FEM is successfully done here with discretisation process.

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