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Evaluation of MSE in Delta Sigma First Order Modulator

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Abstract- The method of Least Square is a technique to point out the best selective line among the corresponding data sequences. In this paper we generalized a virtual structure as well as truthful block diagram of first order adaptive delta sigma modulator. It is shown how the MSE algorithm can be utilized in delta sigma modulation. Simulation result show that the performance of this process for various types of tested signal in case of reducing the Signal to Noise Ratio

Key Words: *Delta Sigma Modulator, Linear Equalizer, Maximum Likelihood Estimator, Decision Feedback Equalizer, MSE algorithm*

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I. INTRODUCTION

Quantization of the amplitudes of discrete signal is in compressed from also introduces some distortion. A commonly used distortion is framed as squared error distortion defined as, f (sampled signal - quantized sample signal) = (sampled signal – quantized sampled signal) ^{2} .

If the above function (f (a_k,a^k)) is measured per sequence is expressed as 'A_n' and the correspon- ding nth quantized value A^n then the function can also be expressed as,

$$f(A_n, \hat{A}_n) = 1/n\sum_{k=1}^{n}(d(a_k, \hat{a}_k)) \quad \text{---------------\dots\dots} \; (1)$$

Now, the equation (1) is sensed as a random variable. Its expected value or mean jointly value is defined as,

$$E\,[f(A_n, \hat{A}_n)] = 1/n \sum_{k=1}^{n} d(ak, a\hat{\ }k) \quad \text{---------\dots\dots} \; (2)$$

Eq^n (2) follows that the total random process with this random variable is stationary.

Now in delta modulation, two level (1bit) quantizer is used with the first order linerar predictive encoder . Here,

Current sampled quantized data = Previous quantized data+ Quantized error data.

Sigma Delta modulator also a predictive type

Quantizer, previously this type of modulator only used where the input sequences are very small but [2] recently in case of large input sequences its used.

The [2] first order delta sigma modulator are combined of a integrator , a uniform quantizer and a negative feedback loop using the integrator and quantizer.

The corresponding diagram is shown below:

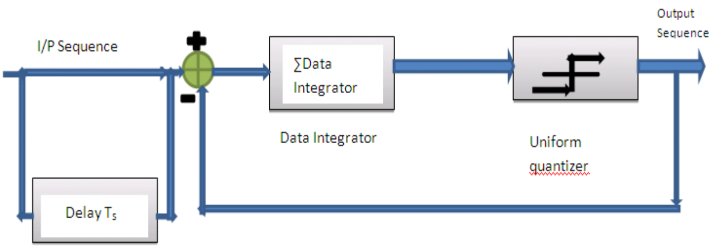

Figure 1. First Order Delta-Sigma Modulator

Here the input sequence connects with one sample delay. If another input sequence in the form of additive white Gaussian noise with one sample delay interprets, spectral energy of the quantization error (difference between noised sampled data and quantized noise sampled data) gives the weight for the high frequency spectrum of most input sequence. Due to this property, our sampling is occurred by this modulator. In this paper, we have to introduce an approach for incremental delta-sigma modulator that happens due to our sampling phenomenon. The new approach allows us to determine the bit intervals of each quantized samples and hence the transfer function of the quantizer. So with this procedure the MSE (Mean Squared Error) is determined for the optimal matched filter. Matched filter is connected to minimize the error probability by maximizing signal-to-noise ratio.

In **Section-II** we study about ISI effects which is inherent to any signal with a finite transmission

bandwidth and its elimination process by some

bandwidth and its elimination process by some equalized filter. **Section-III** we established the least mean square algorithm process to a delta modulator's output sequence with an expression from quantisation error sequence.

II. ISI ELIMINATION :

In this paper at the time of designing the optimum filter in modulator we experience a distortion which results ISI due to which in one span of period the two different types of spectra will be produced. The solution [3] of ISI problem is

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done through designing a compensator which is an equalizer.

It is mainly of three types-

(i) Linear Equaliser
(ii) Maximum Likelihood Sequence Estimation (MLSE)
(iii) Decision Feedback Equalisation

(i) <u>Linear Equaliser -</u>

It's often used by modulator (called transversal) arrangement.

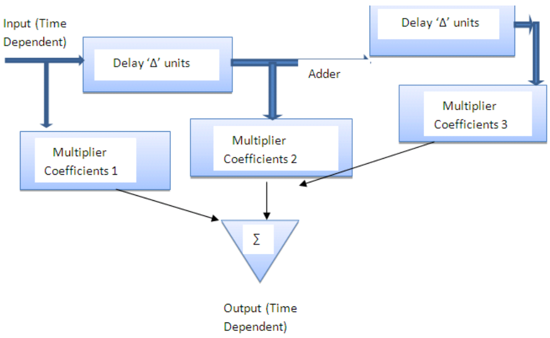

Figure 2. Linear Type Equalizer

Now,

Output(t) = (Multiplier Coefficients) * (Input(t)) + (Multiplier coefficients) * Input (with 'Δ' Delay).

If the coefficients are unique and unit, then the frequency dependent Output (By Fourier Transform of Output (t))
That is, say,

$$Y(f) = \{1+\exp(-j\omega\Delta)\}\ X(f) \ \ldots\ldots. (3)$$

where Y(f) and X(f) are the frequency dependent O/P and I/P respectively. The equation (3) suggests that the O/P samples are the filtered version of the I/P samples with the filter function H(f).

Now,

$$H(f) = \exp(-j\omega\Delta/2)\{\exp(j\omega\Delta/2)+\exp(-j\omega\Delta/2)\}$$

$$\text{or, } H(f) = 2\exp(-j\omega\Delta/2)\cos(\omega\Delta/2)$$

$$\text{or, } |H(f)| = 2\cos(\pi/\Delta) \text{ which is LPF} \ldots\ldots\ldots\ldots. (4)$$

and of the multiple coefficients are equal but of opposite polarity, then

$$|H(f)| = 2\sin(\pi/\Delta) \ \ldots\ldots. (5)$$

which is HPF.

(ii) <u>MLSE:</u>

It has a computational complexity that grows exponentially as M^{L+1} where M is the size of symbol alphabet and 'L' is the number of interference symbols interpreted to the ISI.

(iii) <u>Decision Feedback Equaliser:</u>

DFE has 2 filters, one each in the forward path and feedback path. The sequence of decisions on previously detected symbol is the input to the filter in the feedback path.

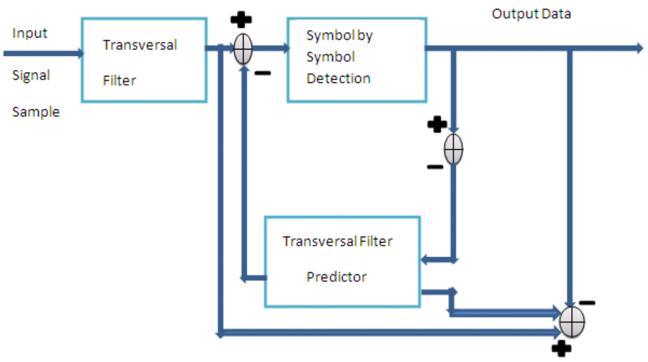

Figure 3. Decision Feedback Equalizer

Now, the performance of the decision feedback equalizer is superior to the linear equalizer when the decision errors are neglected and may be attributed to the inclusion of the feedback section.

The performance MLSE compared with DFE is superior but that is obtained at the cost of large computational complexity. A better way to keep track of the changing medium characteristics is to use adaptive equalizer incorporating microprocessor for automatic rather than tapped gain adjustments which works on Least Mean Squared Error Algorithm.

III. Least Mean Square Algorithm Effects on Delta Sigma Modulator:

Let us consider a zero facing equalizer described with (2N+1) tap gains. The output of the equalizer at the ith instant maybe written as,

Output(i) = $\sum_{i=-N}^{N} (Nth\ tap\ gain\ coefficients)$ *
Input(i=N) ………………………….. (6)

which differs from transmitted signal, x(i)

Now, $E[\{output(i)-x(i)\}^2$

$=\sum$[(centroids - length of the quantised bit sequenced interval) – (quantized interval provided by the linear filter)] …………………………………………… (7)

where $\partial E / \partial(tap\ gain\ coefficients) = 0$ ……... (8)

E is the Expectation of the Random Variable.

Now, the characteristics of linear equalizer is measured by using MSE algorithm that is mean square error algorithm and also the corresponding probability of error is determined.

One approach to compute this probability of error is Brute Force Algorithm [5] which is illustrated below.

Step 1: Set a PAM signal having symbol say, 2p-N-1, p = 1,2…N, with the equally likely probability.

Step 2: Now, after taking selection on the symbol say,

S_p .

Now, its quantized value say, $\hat{S}_p$.

$$\hat{S}_p = h_o S_o + \sum_{\substack{i!=0\\ p=1}}^{N} S_j h_{p-j} + \sum_{j=-i}^{i} r_j \eta_{p-j} \ \ldots\ldots\ldots\ldots\ldots (9)$$

where, 'h_o' is the impulse response of equalizer and the chl. The first term of equation (9) in RHS, combining ISI in the form of middle term in RHS of equation (9) with which last term is the Gaussian noise (AWGN) (η).

Here for, converting at first analog continuous to discrete form we select PAM signal as a reference.

Step 3: Now, the probability of error for an unchanged ISI ($€_j$)

is, $P_N(€_j)$ = Twice * ((N-1)/N) $P(\eta+€_j>h_o)$

= Twice * ((N-1)/N) $Q((h_o-€_j)^2/(variance)^2)^{0.5}$ …(10)

The best quanta among the samples from the PAM signal is given by,

$$i_m = \int_{iM} p f(p)\, dp \,/ \int_{iM} p f(p)\, dp \ \ldots\ldots\ldots (11)$$

Now, it is very complex to directly apply the MSE or LMS algorithm directly in delta modulation rather delta-sigma modulation. Solve will try to work with quantisation noise error sequence in delta-sigma modulator.

The h(n), impulse response is zero for all n<0 if h(n) is causal. Then for n>a (a should be a minimum value starting from -∞) the error sequence,

$$e(n) = \sum_{i=0}^{\infty}[h(i) - h(i-1)]\, q(n-i) \ \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where, q(n) denotes the quantisation sequence

Now, Actually [4] MSE is a technique for evaluation of average noise power by which the overall accuracy of the system is sustained through linear filter. In plotting the MSE, the linear filter is chosen for its systematic offset and gain biases using the minimum MSE is taken by computing the co-efficient 'α' , 'β' minimize.

$$D = E[(u-\hat{u})^2] = \sum_{i=1}^{\gamma} (\alpha u_i - \alpha \hat{u}_i - \beta)^2\, \Delta_i$$

$$= \sum_{i=1}^{\gamma} \alpha(q(i)-\beta)^2\, \Delta_i \ \ldots\ldots\ldots (13)$$

$\{\hat{u}\}_{i=1}^{\gamma}$ and $\{\Delta_i\}_{i=1}^{\gamma}$ are the centroids and length of the quantization step size respectively.

Now the quantized signal , q(i) [2] must follow a distribution function which is given below:

$q(i) = i + \Delta_i/2$, ………………….. if $(-\gamma \le i < \gamma)$

$= \gamma - \Delta_i/2$, …………………… .. if $i > \gamma$

$= -\gamma + \Delta_i/2$ …………………….. if $i < -\gamma$ …….. (14)

γ is a integer multiple of Δ_i

Consider this $\gamma = \alpha^2_k/\eta_k$

Here, γ is exponentially distributed, α_k is Rayleigh distributed.

$$\alpha^2_k / \eta_k = \frac{\text{Signal Power}}{\text{Noise Power}}$$

IV. Conclusion

As we have come to the end of this report we can clearly see that the noise power is decreased to some extent with the use of least mean square algorithm. The output of the programs is obtained by using MATLAB software. From the graph of LMS adaptive equalization the space between the consecutive quantas decreases and as a result the error reduces

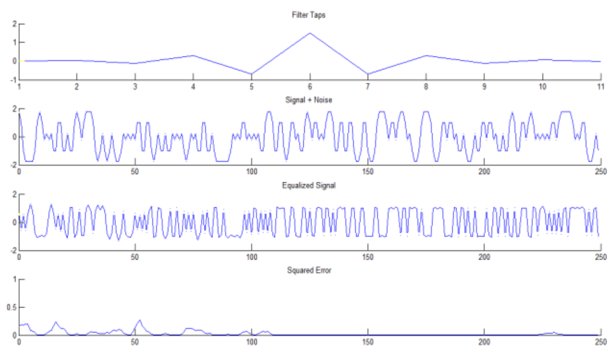

Figure 4 : Effect of Noise in Adaptive Equaliser

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The simulink waveform of first order delta sigma filter for various type of input signal is shown below :-

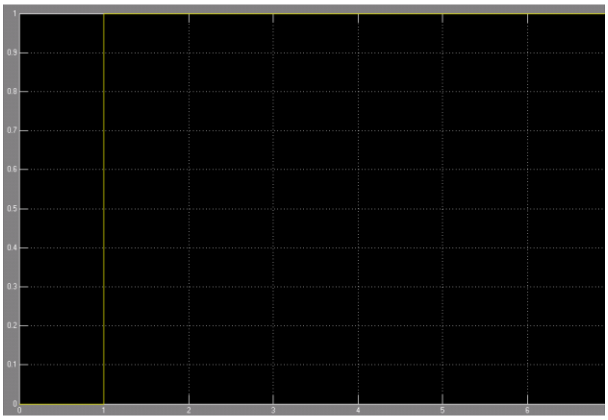

Figure 5(a) : First Order Delta Sigma Modulator for Step signal input

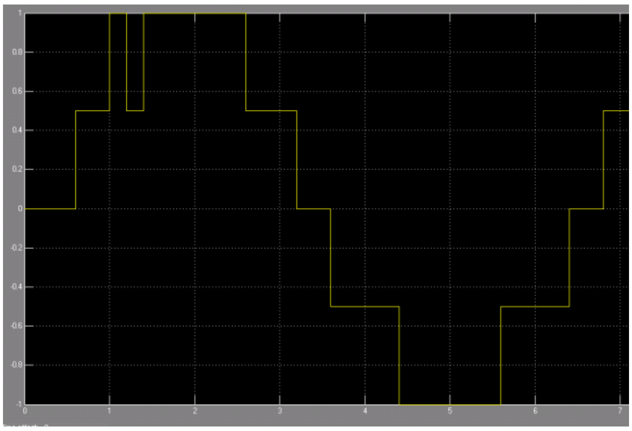

Figure 5(b) : First Order Delta Sigma Modulator for Sinusoidal signal input

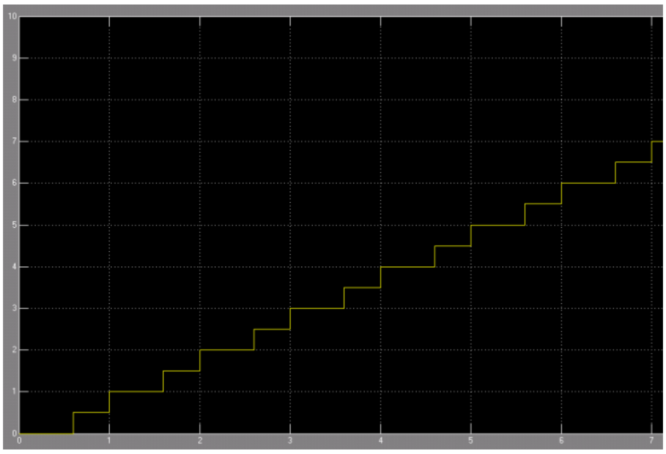

Figure 5(c) : First Order Delta Sigma Modulator for Ramp signal input

Now if the modulator is experinced with an additive Gaussian Noise, then the corresponding diagrams are shown below :

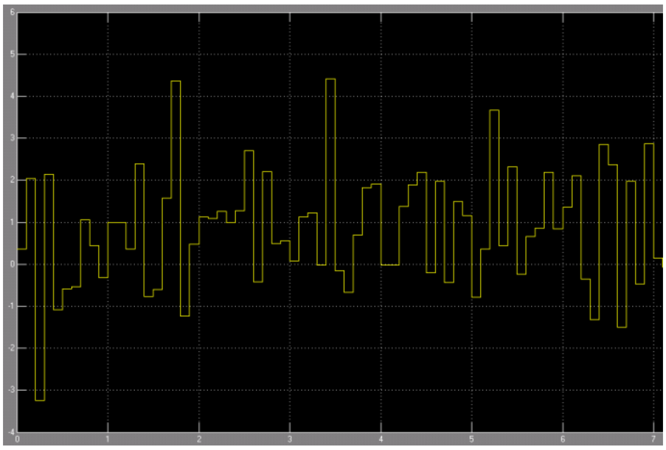

Figure 6(a) : First Order Delta Sigma Modulator with AWGN for Step signal input

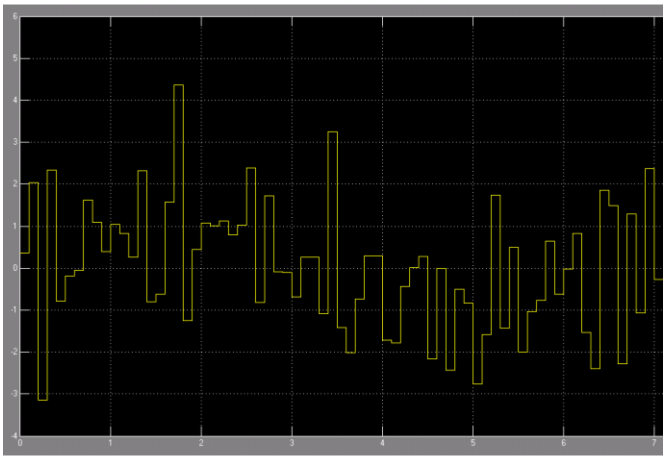

Figure 6(b) : First Order Delta Sigma Modulator with AWGN for Sinusoidal signal input

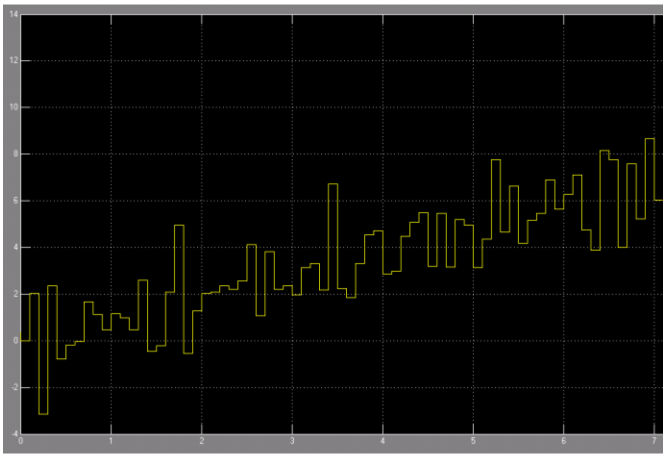

Figure 6(c) : First Order Delta Sigma Modulator with AWGN for Ramp signal input

Now if the quantized signal follow the Rayleigh Distributed function as stated earlier , then using MSE we see that the Signal to Noise ratio is improved by increasing the Number of sequences which is denoted in the following figrure by 'N'.

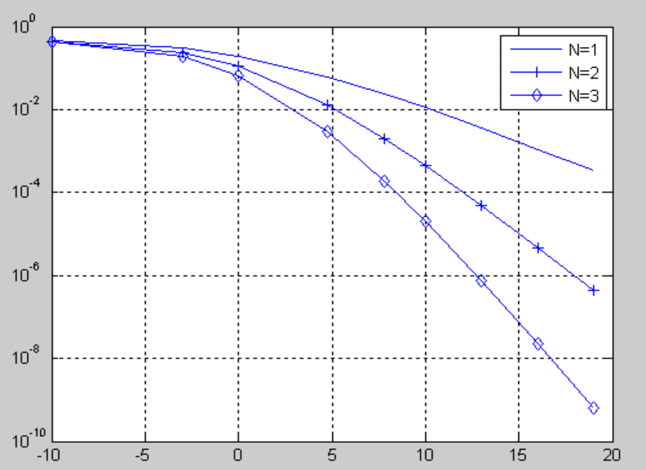

Figure 7 : Signal to Noise Ration vs γ Random Variable

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