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Capacity Improvement in Multi-User MIMO System using Dirty Paper Coding

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Abstract—The capacity of Gaussian broadcast MIMO channel can be improved with Dirty Paper Coding. In this paper, the mean capacity for MU-MIMO (Multi-User Multiple Input, Multiple Output) is analyzed in terms of number of frames, available number of users at mobile station and number of packets. The analysis is done for 4x4 matrix. Dirty Paper Coding improves the degradation due to correlation and mutual coupling. In multi-user communication, multiple antennas allow users to transmit their data stream in the uplink and downlink

Keywords- *Multiple Input Multiple Output (MIMO), Broadcast Channel(BS), Multiple access Channel (MAC), Multi User Multiple Input Multiple Output (MU-MIMO), Dirty Paper Coding (DPC), Signal correlation, Mean capacity.*

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I. INTRODUCTION

In conventional communication system, single antenna is used at two sides of communication link. In wireless communication system, multiple transmitting and receiving antennas are used for achieving greater system capacity [1]. The system capacity is bounded by correlation [2]. Correlation is due to closeness of antenna elements [3]. In multi path channel, the MIMO capacity for single user $N_R x N_T$ system is proportional to $N_{min}=min(N_T,N_R)$ [4]. In single user MIMO system, a point to point high data rate transmission can be supported by spatial multiplexing while providing spatial diversity gain. In modern communication era, most communication system deals with multiple users, sharing the same radio sources [5]. For enjoying applications such as Wireless LAN, Cellular telephony, single base station must communicate with many users simultaneously. Therefore, the study of Multi-User MIMO systems has emerged as an important research topic recently [6].

In Figure 1. Multi-user communication is shown. The mobile stations are served by a single base station.

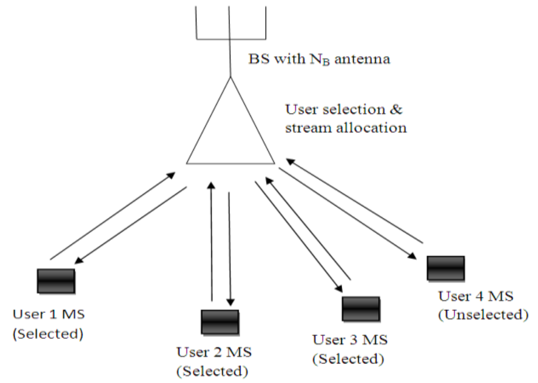

Figure1. A Multi-user MIMO system for K=4

Here, four users (K=4) are connected to base station. Three users are selected and allocated communication resource such as time, frequency, and spatial stream. Base station and each mobile station are equipped with N_B and N_M antennas. In multi-user, K.NM antennas can communicate with a single base station (BS) with N_B antenna. So, in multi-user MIMO system, multiple independent users can transmit data to base station (BS) and can decode data by each user simultaneously. Multi-user MIMO is attributed to increase in degree of freedom with multiple antennas as in the single user MIMO system [7]. Multi-user MIMO requires perfect data and power cooperation between the base stations. The sum capacity in a Multi-user MIMO is equivalent to maximum aggregation of user's data rates.

II. MATHEMATICAL MODEL FOR MULTI USER MIMO [7]

A. Uplink Channel (Multiple access channel)

In Figure 2. Uplink channel is mathematically modeled. As discussed earlier, base station and mobile station are equipped with N_B and N_M antennas. The received signal for uplink is given by

$$y_{MAC} = H_1^{UL} \chi_1 + H_2^{UL} \chi_2 + \dots\dots\dots H_K^{UL} \chi_K + z \qquad (1)$$

$$= \left[H_1^{UL} \quad H_2^{UL} \quad \ldots\ldots\ldots\ldots \quad H_K^{UL} \right] \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} + z$$

$$= H^{UL} \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} \qquad (2)$$

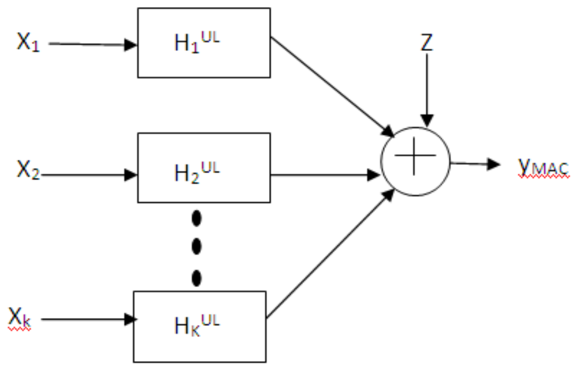

Figure 2. Uplink channel model for MU-MIMO [7]

B. Downlink Channel (Broadcast Channel)

The received signal in downlink is given by

$$y_u = H_u^{DL} x + z_u \quad \text{where } u=1,2,3\ldots\ldots K \qquad (3)$$

The overall system of downlink is represented by following equation

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ \vdots \\ H_K^{DL} \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix} \qquad (4)$$

Equation (4) shows mathematical expression for downlink channel.

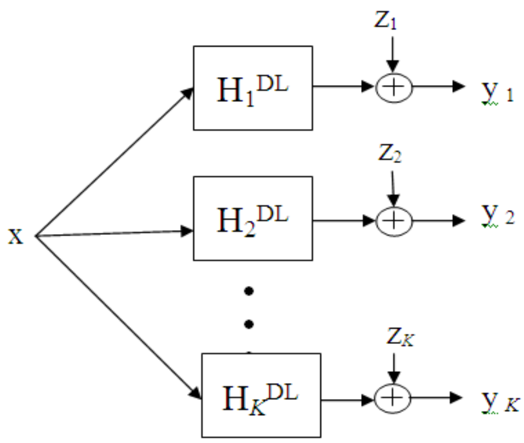

Figure 3. Downlink channel model for MU-MIMO [7]

III. CAPACITY ANALYSIS OF BROADCAST CHANNEL USING DIRTY PAPER CODING [7]

Downlink capacity is defined as the maximum aggregation of all users' data rates. Downlink capacity for two base station antennas, $N_B = 2$, one mobile station antenna, $N_M = 1$ and one user, K=1 using Dirty Paper Coding is expressed as follows. In this case the received signal is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (5)$$

Here H^{DL} denotes the channel matrix between base station and u^{th} user where, u=1, 2. x_1 and x_2 is transmitted signal by two base station antennas. If the channel information is completely available at base station, the overall channel can be decomposed as

$$H^{DL} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad (6)$$

Where

$$l_{11} = \left\| H_1^{DL} \right\|, \quad q_1 = \frac{1}{l_{11}} H_1^{DL},$$

$$l_{21} = q_1 \cdot \left(H_2^{DL} \right)^H,$$

$$l_{22} = \left\| H_2^{DL} - l_{21} q_1 \right\|,$$

$$q_2 = \frac{1}{l_{22}} \left(H_2^{DL} - l_{21} q_1 \right),$$

Equation (5) is the orthonormal row vectors. From the channel information, pre-coded transmitted signal is given by equation (7)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Q^H \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \frac{1}{l_{22}} l_{21} \tilde{x}_1 \end{bmatrix} \qquad (7)$$

By transmitting the above pre-coded signal

$$y_{BC} = H^{DL} x + z \qquad (8)$$

$$= \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \begin{bmatrix} q_1^H & q_2^H \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \frac{1}{l_{22}} l_{21} \tilde{x}_1 \end{bmatrix} + z \qquad (9)$$

$$= \begin{bmatrix} l_{11} & 0 \\ 0 & l_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + z \qquad (10)$$

$$= \begin{bmatrix} \|H_1^{DL}\| & 0 \\ 0 & \|H_2^{DL} - l_{12} q_1\| \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + z \qquad (11)$$

From equation (9), (10), and (11), it is observed that two virtual interference free channels are created. Here, total power is divided into αP and (1-α)P for the first and second users.

$$E\{|x_1|^2\} = E\{|\tilde{x}_1|^2\} = \alpha P \qquad (12)$$

$$E\{|x_2|^2\} = E\left\{\left|\tilde{x}_2 - \frac{l_{21}}{l_{22}}\tilde{x}_1\right|^2\right\} \qquad (13)$$

Equation (12) and (13) is equal to αP and (1-α)P. Then the capacities for the first and second user are given as

$$R_1 = \log\left(1 + \|H_1^{DL}\|^2 \frac{\alpha p}{\sigma_z^2}\right) \qquad (14)$$

$$R_2 = \log\left(1 + \|H_2^{DL} - l_{21} q_1\|^2 \frac{(1-\alpha P)}{\sigma_z^2}\right) \qquad (15)$$

If the second user is selected such that $l_{21} = 0$, then capacity will be represented by equation (16)

$$R_2 = \log\left(1 + \|H_2^{DL}\|^2 \frac{(1-\alpha P)}{\sigma_z^2}\right) \qquad (16)$$

Remaining values of third and fourth antenna capacities may also be calculated.

IV. IV. ANALYSIS OF DIRTY PAPER CODING

Dirty paper coding is a coding technique that pre-cancels known interference without power penalty. Only transmitter needs to know this interference, but full channel state information is required everywhere to achieve the weighted sum rate Dirty Paper Coding. Dirty Paper Coding technique requires knowledge of the interference state in a non-causal manner [8]. The design of a DPC-based system should include a produce to feed side information to the transmitter. Interference free transmission can be realized by subtracting the potential interferences before transmission. The working of Dirty Paper Coding can be explained by Figure 4.

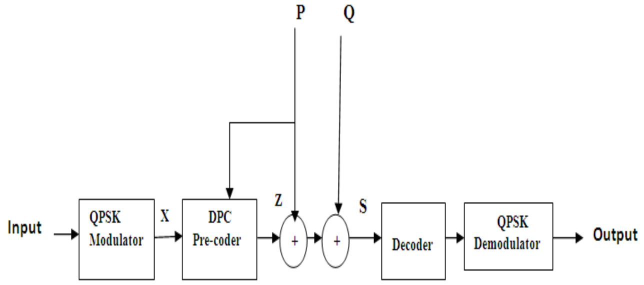

Figure 4. Communication system model using Dirty Paper Coding

The Dirty Paper Channel model is depicted in Figure (4). The received signal for such system is given by

$$S = Z+P+Q \qquad (17)$$

Where, P is arbitrary interference known at transmitter, N is statistically independent Gaussian random variable. If P, the known interference subtracted at receiver, interference poses no problem. Similarly known interference subtracts at transmitter, and then transmitted signal is

$$Z' = Z - P \qquad (18)$$

Now the received signal is given by

$$S' = Z' + P + Q \qquad (19)$$

$$S' = Z - P + P + Q$$

$$= Z + Q \qquad (20)$$

When Dirty Paper Coding is applied in the MIMO system, mathematically it will be equal to scaled inverse matrix of lower triangular matrix.

V. SIMULATION AND DESIGN

For simulation 4x4 matrix is considered. There are 4 Base station antennas and 4 Mobile stations are taken into consideration. Quadrature phase shift keying modulation scheme is used for transmission. Here, the λ/2 ideal dipole is used for analysis [9]. Perfect channel state information at the receiver (CSIR) are considered for the analysis.

NO. OF FRAMES	10
NO. OF PACKETS	250
NO. OF BASE STATION(BS) ANTENNA	4
NO. OF MOBILE STATION(MS) ANTENNAS	4
NO. OF USERS	10,20,30

VI. RESULTS

Mean capacity versus signal to noise ratio (SNR) is analyzed with Dirty Paper Coding and without Dirty Paper Coding. The simulation is done between 0 to 20 dB and 5000 iterations are taken.

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As observed from simulation results, at signal to noise ratio 20dB, mean capacity has been improved as compared to without Dirty Paper Coding for 10, 20 and 30 users.

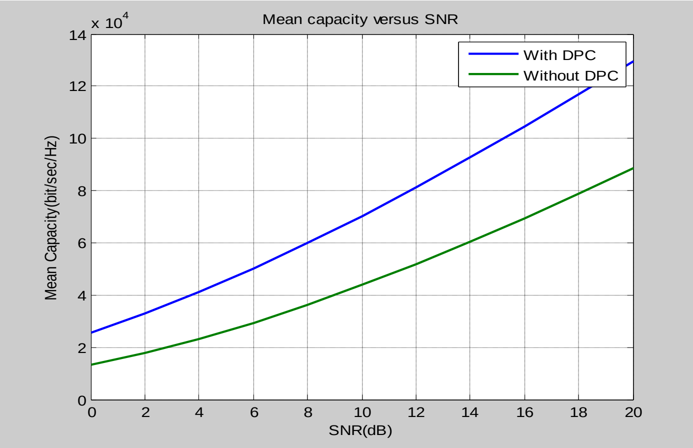

Figure 5. Mean capacity versus SNR when number of users are 10 .

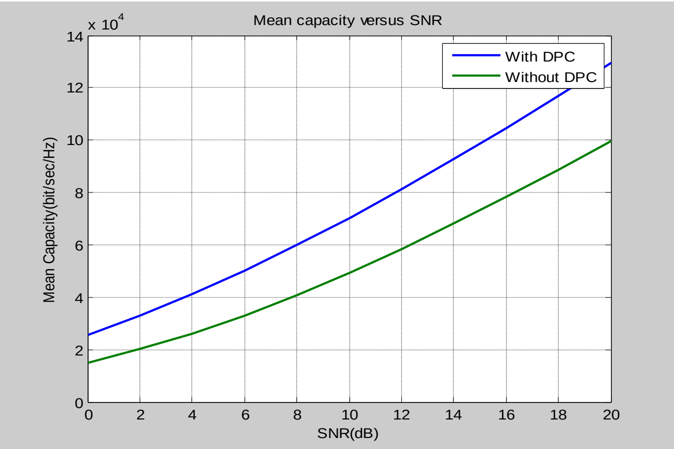

Figure 6. Mean capacity versus SNR when number of users are 20.

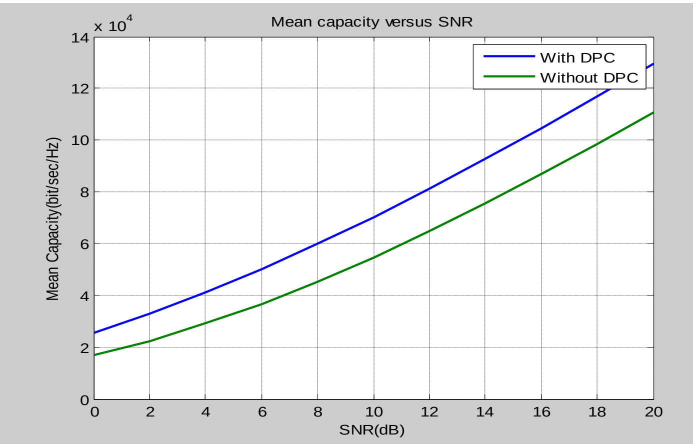

Figure 7. Mean capacity versus SNR when number of users are 30.

VII. CONCLUSION

From the analysis, it is concluded that Dirty Paper Coding improves the capacity of MU-MIMO system compared to MU-MIMO system without Dirty Paper Coding.

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