Accelerated Target Detection using Fractional Fourier Transform

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Abstract – Relative motion between the radar and target characterize the radial acceleration of the radar target, which is considerable for recognition and tracking of target. The echo of target is considered as Linear Frequency Modulated (LFM) or chirp signal due to its radial acceleration. Fractional Fourier Transform (FrFT) is the optimum method to estimate the parameter of LFM signal because it concentrates the energy of the chirp signal in the form of an impulse at proper order. In this paper the model of accelerated target is purposed and the acceleration of target is estimated using FrFT. Anti-noise interference capability of FRFT in estimation of acceleration is studied with reference to WVD-HT (Wigner-Ville Distribution and Hough Transformation) and WVD (Wigner-Ville Distribution). The MATLAB simulations were made to prove that estimation of acceleration using FrFT was better than other methods.

Keywords*- Fractional Fourier Transform (FrFT), chirp signal, parameter estimation, MATLAB.*

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I. INTRODUCTION

Linear frequency modulated (LFM) signal is widely used for radar system, acoustic communication and sonar system [1]. In a noisy environment detection and estimation of the LFM signal are extremely important, and they gain considerable attention in recent years. Radar transmitted signal is modulated as LFM signal due to the relative motion between radar and target, and nonlinearity exists due to the acceleration of target. Non linearity in the signal makes the spectrum aberrant. Target detection using a conventional FFT (Fast Fourier Transform) decreases the performance due to nonlinearity. The aberrant spectrum contains the information of radial acceleration. In the field of military, radar did not provide acceleration information because in the early time aircrafts have low mobility. In these days with the help of advancement in technology, the mobility of aircrafts have increased, therefore the effect of acceleration on signal spectrum of FFT could not neglect.

There are several methods for extracting the time-varying characteristics of signal, among them simplest method is the STFT (Short Time Frequency Transform). STFT uses sliding window function to extend Fourier Transform, but the narrow or time-variant window results a poor resolution in the time-frequency domain [2]. Wigner Distribution was introduced by J.Vill in the signal processing field [3]. Wigner-Ville Distribution (WVD) could be used for moving target detection and parameter estimation, but WVD suffer from the interruption of cross-terms in the presence of multi-component signals [4]. Although the effect of interference can be suppressed by carefully selecting the kernel function of TFD, but meanwhile, their time-frequency localization performance will be certainly degraded [5]. Later on the method for analysis of the multi-component LFM signal is presented based on the WVD-HD (WVD-Hough Transform) [6]. This method has the capability to eliminate the disturbance induced by the cross-terms, by means of an integral over a line in the time-frequency plane. The main drawbacks of this method lie in the computation time of the WVD-HT and during the transformation the initial phase of signal is lost therefore it cannot be approximated from the detection statistics. Wigner-Ville distribution was used to approximate radial acceleration of target [7, 8].

Radial acceleration with radar and target echo with -10dB Signal-to-noise (SNR) was calculated, that requires a long duration signal (341 ms). WVD-HT was used to detect and estimate the parameters of radially accelerated target [9]. Along with the advancement of Time-Frequency method, multi-component chirp signal detection and parameter estimation technology based on different time-frequency analysis tools appears ceaselessly, including discrete Chirp-Fourier transform (DCFT) [10]. Performance of parameter estimation using DCFT is good, but it requires the modulated slope is an integer. Detection and estimation using FrFT compared with the WVD and WVD-HT has lower computational complexity and computational speed is also reduced clearly.

In this paper radial acceleration of target is estimated using Fractional Fourier Transform (FrFT) for different time periods and different SNR values. Using this technique, it can be seen that the method has good precision and accuracy and it also improve the computation speed. On the other side, its anti-noise interference capability is better than any other method. To verify these characteristics, a mathematical model for target with uniform acceleration is deduced and then the computational formula is established. In addition, the relation between accuracy of estimation with other parameters was studied. Finally simulations were conducted using MATLAB computational platform and the results are verified.

II. ESTIMATION OF TARGET ACCELERATION USING FrFT

A. Mathematical model of accelerated target

The signal transmitted by radar can be considered as given below:

$$s(t) = A\cos[2\pi f_0 t + \emptyset_0] \quad 0 \le t \le T \tag{1}$$

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where, A, f_0 and $\emptyset_0$ represent amplitude, starting frequency and phase of transmitted signal respectively.
There is a relative motion between target and radar. To simplify the solution, assume that the target is moving towards the radar. As shown in the given figure 1, the target in the given time period is seen as linear motion of uniform acceleration.

When the transmitted signal hit the target, the received target echo is considered as:

$$R(t) = A\exp\left(j\left(2\pi(f_0 t + f_d t + k/2\, t^2) + \phi'_0\right)\right) \quad 0 \le t \le T \qquad (2)$$

where, $f_d = \frac{2v}{\lambda_0}$ is the Doppler frequency.
It can be noticed that the square term in (2) is due to acceleration that varies as square with time.
The chirp rate or frequency modulated parameter k has relation with acceleration 'a' as given below:

$$k = \frac{2a}{\lambda_0} \qquad (3)$$

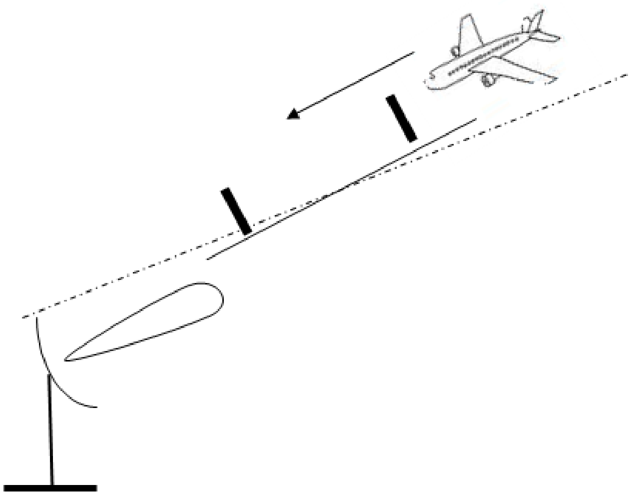

Fig. 1 Target moving towards the radar with uniform acceleration

The instantaneous frequency of received echo signal is:

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = f_0 + f_d + kt \qquad (4)$$

The reflected signal R(t) from target add up with Additive White Gaussian Noise (AWGN) in the free space and the noisy signal is given by [9]:

$$y(t) = A\exp\left(j\left(2\pi(f_0 t + k/2\, t^2) + \phi'_0\right)\right) + w(t) \qquad (5)$$

where, Gaussian noise is represented by $w(t)$.

B. *Definition of FrFT*

It can be seen from analysis that, the reflected signal from target is a chirp (LFM) signal. There are number of techniques to analyze the LFM signal in the Time-Frequency domain. In 1980, FrFT technique is given by Namias in the field of quantum mechanics, and this is used in this paper to estimate the acceleration of target. After that, Almeida find out the relation between FrFT and WVD, and also discuss about the rolling operator in time –frequency plane. This property of rolling factor is applicable in LFM signal analysis.
FrFT is the generalization of conventional Fourier Transform (FT) and it can rotate the signal coordinate in counter clockwise direction about the origin in Time – Frequency domain. By operating a signal with FT, the signal can be observed as $\frac{\pi}{2}$ anti-clockwise rotation from the time axis to the frequency axis, similarly FrFT can be observed as anti-clockwise rotation from the time axis to the u axis with an angle α.

The FRFT of given signal y(t) is considered as:

$$Y_\alpha(t,u) = F^p[y(t)] = \int_{-\infty}^{\infty} y(t)\, K_\alpha(t,u)\, dt \qquad (6)$$

where, p is called the order of FRFT and $K_\alpha(t,u)$ is the kernel of FRFT:

$$K_\alpha(t,u) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}}\exp\left(j\frac{t^2+u^2}{2}\cot\alpha - tu\csc\alpha\right), & \alpha \ne n\pi, \\ \delta(t-u), & \alpha = 2n\pi, \\ \delta(t+u), & \alpha = (2n+1)\pi, \end{cases} \qquad (7)$$

where, $\alpha = p\frac{\pi}{2}$

Applying the optimum order FrFT the signal energy is concentrated in the bandwidth of $B_m = |2\sin\alpha/T|$ and the peak value is $|X_p(u)|^2$, if and only if $k + \cot\alpha = 0$ and $f_0 - u\csc\alpha = 0$. As per the above property, estimation of the signal y(t) parameters can be derived as [2]:

$$\{\hat{p},\ \hat{u}\} = \mathrm{argmax}_{p,u}|Y_p(u)|^2 \qquad (8)$$

$$\begin{cases} \hat{k} = -\cot(\hat{p}\pi/2) \\ \hat{f_0} = \hat{u}\csc(\hat{p}\pi/2) \end{cases} \qquad (9)$$

Dimensional normalization is required before applying the discrete algorithm of FrFT [11], which means to describe the corresponding analysis range and discrete resolution in the FrFT domain. In practical discrete signal, the discrete scaling transform method is mostly used [12]. After applying the discrete scaling transform method, (9) have to be adjusted as:

$$\begin{cases} \hat{k} = -\frac{f_s}{t_d}.\cot(\hat{p}\pi/2) \\ \hat{f_0} = \sqrt{\frac{f_s}{t_d}}.\hat{u}\csc(\hat{p}\pi/2) \end{cases} \qquad (10)$$

where, f_s and t_d represents the sampling frequency and sampling duration respectively.
The target acceleration estimation is based on the two dimensional search. According to (8) the peak position is at $\hat{u}$ and $\hat{p}$ is the estimated FrFT order. Using that value the acceleration estimation formula is derived as:

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$$\hat{a} = \frac{\lambda f_s}{2T}\cot(\hat{\alpha}) \quad (11)$$

where, $\hat{\alpha} = \hat{p}\,\frac{\pi}{2}$

III. SIMULATION RESULTS AND ANALYSIS

A. *Estimation of radial acceleration with different value of SNR*

In term to estimate the parameter of accelerated target, it is considered that the initial speed of target is 45 m/s, acceleration is 200 m/s^2, the radar wavelength λ = 20 mm, duration of received signal is 100ms and the sampling frequency is 20 KHz.

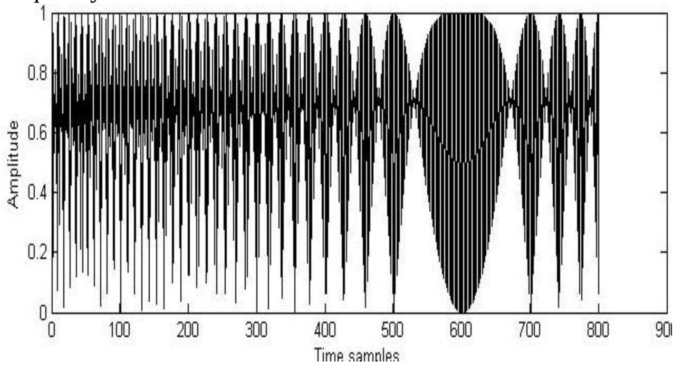

Fig. 2: Radar received chirp signal without noise.

According to (8), the estimated value of FrFT order and position of peak are 0.9368 and 0.0231 respectively.
To estimate the acceleration, simulations are conducted when SNR varies from -15 to 0dB. After averaging 100 times, the result showed below in Table 1.

TABLE1. Acceleration estimation for negative value of SNR

SNR(dB)	Acceleration (m/s^2)
-15	201.67
-10	197.00
-8	196.98
-6	201.243
-4	196.98
-2	200.85
0	199.21

Now SNR value is taken from 2 to 10dB and the simulation results is given below in table 2. In [9], WVD-HD is used to estimate the acceleration. The estimated value is deviated from the actual value by 1.087 %, SNR was -15dB. In Table 1 for -15dB, it is shown that the precision is improved.

TABLE2. Estimation of radial acceleration for SNR 2 to 10dB

SNR(dB)	2	4	6	8	10
Acceleration (m/s^2)	200.90	200.71	200.85	200.91	200.91

B. *The result of estimation for different signal duration:*
To estimate the parameter of accelerated target at different signal duration, it is considered that the initial speed of target is 45 m/s, acceleration is 200 m/s^2, the radar wavelength λ = 20 mm, duration of received signal is 100ms and the sampling frequency is 20 KHz and the SNR is 10dB. The results of simulation are shown in Table 3.

TABLE 3 Estimation of Acceleration at different window size for received signal

T(s)	Acceleration (m/s^2)	Peak Magnitude
0.1	199.27	16.71
.09	200.79	2.10
.08	199.84	1.76
.07	201.54	1.50
.06	199.28	1.28
.05	199.12	1.35
.04	334.21	1.20
.03	445.00	1.20

In Table 3, it can be observed that the precision of measurement depend upon the duration of signal. The signal of longer duration has better precision than the signal of short duration. The amplitude of impulse peak also depend upon the signal duration, energy of shorter signal is less as compare to the longer duration signal therefore the peak amplitude is small.

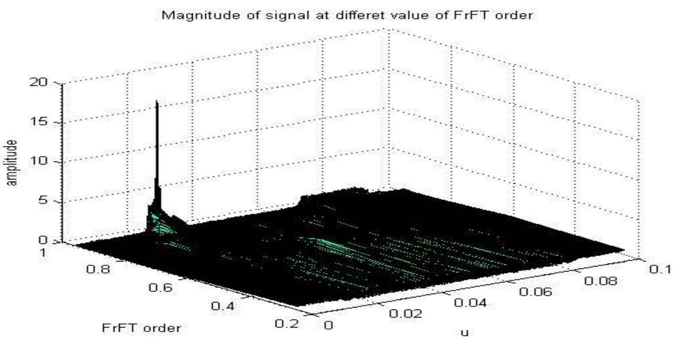

Fig. 3 Energy distribution of received signal at different order of FrFT.

From the simulation, received echo signal FRFT for the different angle rotation is shown in Figure 3 and the impulse is obtained at rotation angle 0.9368π/2 i.e. energy concentrated at that angle.

IV. ACCURACY ANALYSIS

To evaluate the estimation error caused due to noise, firstly an observation signal is generated which is modeled as (5) with parameters A=1, f_0 = 200*Hz*, and K= 200 *Hz/s*. Then for the above given parameters the estimation value is computed (10) for 100 times. Now the accuracy of these estimated parameters is given by root mean square error (RMSE), given as:

$$\text{RMSE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi}-\xi\right)^2} \quad (12)$$

Where, $\widehat{\xi_i}$ depicts the estimated value ξ in the i^{th} iteration, and N is the total number of iteration. The value of N is 100 in this simulation. The normalized value of RMSE is given as:

$$\text{Normalized RMSE} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi}-\xi\right)^2}}{Max\left(\hat{\xi}\right)-\min\left(\hat{\xi}\right)} \quad (13)$$

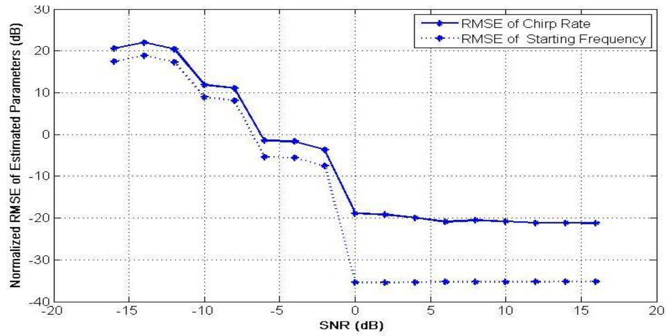

Fig. 4 shows the Normalized RMSE estimated parameters K (Chirp rate) and f_0 (Starting frequency) under various SNR

V. CONCLUSION

In this paper, the echo of the radially accelerated target is estimated. The algorithm to estimate the parameters of radial accelerated target was brought forward. It is analyzed that the FrFT is good enough to estimate the parameters. The analysis shows relation between estimated acceleration and duration of signal. Accuracy of estimation is inversely proportional to the duration of received signal. Energy distribution of signal at different rotation angles is shown. At last the accuracy of estimation of initial frequency and chirp rate at different SNR value was determined. Simulation results proved that this method will be able to improve the operational speed, control operational precision and give better anti-noise interference capability then other scheme.

VI. REFERENCES

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