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A Note on Generating Functions and Summation Formulae for Modified Generalized Sylvester Polynomials

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Abstract: The present paper deals with three different classes of generating functions and various elegant summation formulae for modified Generalized Sylvester polynomials.

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1. INTRODUCTION:

Generalized functions occupy the pride of place in literature on special functions. Their importance which is mounting everyday stems from the fact they generalize well-known one variable special functions namely Hermite polynomials, Laguerre polynomials, Legendre polynomials, Gegenbauer polynomials, Jacobi polynomials, Rice polynomials, Generalized Sylvester polynomials etc. All these polynomials are closely associated with problems of applied nature. For example, Gegenbauer polynomials are deeply connected with axially symmetric potentials in n dimensions and contain the Legendre and Chebyshev polynomials as special cases. The hypergeometric functions of which the Jacobi polynomials is a special case, is important in many cases of mathematics analysis and its applications. Further, Bessel functions are closely associated with problems possessing circular or cylindrical symmetry. For example, they arise in the study of free vibrations of a circular membrane and in finding the temperature distribution in a circular cylinder. They also occur in electromagnetic theory and numerous other areas of physics and engineering.

We have defined the modified generalize Sylvester polynomials $f_n\ (x;a,b)$ as follows $(see$ [10]):

$$f_n\ (x;a,b) = \frac{(bx)^n}{n!}\,{}_2F_0\left[\begin{matrix} -n; & ax; & \\ & & \frac{-1}{bx} \\ & -; & \end{matrix}\right].. \qquad .(1.5)$$

Where $b \neq 0$ is an arbitrary constant .

When $a = 1$ and $b = 1$ then (1.5) becomes

$$f_n\ (x;1,1) = \Phi_n(x) \qquad (1.6)$$

We call the polynomials $f_n\ (x;a,b)$ modified generalized Sylvester polynomials in view of the relations (1.6). For $a = 1$ and b by a (1.5) becomes A. K. Agarwal and H.L.Manocha [1] generalization of Sylvester polynomials.

The following generating relations hold for (1.5)

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$$\sum_{n=0}^{\infty} f_n\,(x;a,b)t^n = (1-t)^{-ax}\,e^{bxt} \tag{1.7}$$

and

$$\sum_{n=0}^{\infty} (\lambda)_n\, f_n\,(x;a,b) = (1-bxt)^{-\lambda}\,{}_2F_0\left[\begin{matrix}\lambda; & ax; & \\ & & \dfrac{t}{1-bxt} \\ & -; & \end{matrix}\right]. \tag{1.8}$$

2. CLASSES OF GENERATING FUNCTIONS:

Let c and d be arbitrary constants. Then the polynomials $f_n\,(x;a,b)$ defined by (1.7) above satisfy the following generating relations:

$$\sum_{n=0}^{\infty} f_n\,(x;a,b+cn)t^n = \frac{(1-u)^{-ax}\,e^{bxu}}{1-cxu}\,, \qquad u = te^{cxu} \tag{2.1}$$

$$\sum_{n=0}^{\infty} f_n\,(x+cn;a,b)t^n = \frac{(1-v)^{-ax}\,e^{bxv}}{1-v[ac(1-v)^{-1}+bc]}\,; \quad v = t(1-v)^{-ac}\,e^{bcv} \tag{2.2}$$

and

$$\sum_{n=0}^{\infty} f_n\left(x+cn;a,\frac{b+dn}{x+cn}\right)t^n = \frac{(1-w)^{-ax}\,e^{bw}}{1-w[ac(1-w)^{-1}+d]}\,; w = t(1-w)^{-ac}\,e^{dw} \tag{2.3}$$

Proof of (2.1):

We know generating function:

$$\sum_{n=0}^{\infty} f_n\,(x;a,b)t^n = (1-t)^{-ax}\,e^{bxt} \tag{i}$$

Expanding the function on the R. H. L. of (i) by Taylor's theorem.

$$\sum_{n=0}^{\infty} f_n\,(x;a,b)t^n = \sum_{n=0}^{\infty} \frac{d^n}{dt^n}\,\{(1-t)^{-ax}\,e^{bxt}\}|_{t=0}\,\frac{t^n}{n!} \tag{ii}$$

Replacing b by $b+cn$ in (ii) we get

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$$\sum_{n=0}^{\infty} f_n\,(x;a,b+cn)t^n = \sum_{n=0}^{\infty} \frac{d^n}{dt^n}\,\{(1-t)^{-ax}\,e^{bxt}\,[e^{cxt}]^n\}|_{t=0}\,\frac{t^n}{n!} \qquad (iii)$$

We know that Lagrange's expansion formula:

$$\frac{f(t)}{1-u\Phi'(t)} = \sum_{n=0}^{\infty} \frac{u^n}{n!} D_t^n \ \left\{f(t)[\phi\,(\mathrm{t})]^n\right\}|_{t=0} \qquad \text{(iv)}$$

Comparing (iii) and (iv) we get (2.1)

3. SUMMATION FORMULAE :

The following summation formulae are easily derivable from known results in view of the relationship (1.7):

$$f_n\,(x;a,b) = \sum_{r=0}^{n} \frac{\{x(b-c)\}^r f_{n-r(x;\,a,b\,)}}{r!}\,, \qquad (3.1)$$

$$f_n\,(x+y;a,b) = \sum_{r=0}^{n} f_{n-r\ (x;a,b)f_r\ (y;a,b)}\,, \qquad (3.2)$$

$$f_n\,(x;a,b+c) = \sum_{r=0}^{n} \frac{(cx)^r f_{n-r(x;\,a,b\,)}}{r!}\,, \qquad (3.3)$$

$$\sum_{r=0}^{n} f_{n-r}(x;a,b+c)f_r(y;a,b+c) = \sum_{r=0}^{n} \frac{\{c(x+y)\}^r f_{n-r(x+y;\ a,b\,)}}{r!}, \qquad (3.4)$$

$$(n+1)f_{n+1}\,(x;a,b) = x\{bf_n\,(x;a,b)\} \sum_{r=0}^{n} f_{n-r\ (x;a,b)f_r\ (y;a,b)}\,, \qquad (3.5)$$

and

$$f_n\,(x;a,bc) = \sum_{r=0}^{n} \frac{\{bx(c-1)\}^{n-r} f_{n\ (x;\,a,b\,)}}{(n-r)!}\,, \qquad (3.6)$$

Proof of (3.1):

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n}\frac{\{x(b-c)\}^r f_{n-r(x;\,a,b\,)}}{r!}t^n = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{\{x(b-c)\}^r f_{n(x;\,a,b\,)}}{r!}t^{n+r}$$

$$=\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{\{x(b-c)t\}^r f_{n(x;\,a,b\,)}}{r!}t^n$$

$$=(1-t)^{-ax}e^{cxt}e^{xt(b-c)}$$

$$=(1-t)^{-ax}e^{bxt}$$

$$=\sum_{n=0}^{\infty}f_n\,(x;a,b)t^n$$

Equating the coefficient of t^n we get (3.1)

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