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A Note on Distributivity of a Poset of Subhypergroup of a Hyper Group

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Abstract: In this paper we defined distributive Property of Poset of Subhypergroup of hyper group and proved this property using principal filter. Also we proved distributivity using translation .we have defined annihilators and ideals of Poset of Subhypergroup of a hyper group. we proved some sufficient condition for Poset to be a distributive Poset.

Keywords:*Poset,Subhypergroup,DistributivePoset,Principlefilter,Annihilator,Ideals,Translations.homomorphism*.

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Introduction

The theory of hyper structures was introduced in 1934 by Marty at the 8th Congress of Scandinavian mathematicians [1]. This theory has been subsequently developed by Corsini[5],Mittas[3]and by various authors, Basic definitions and propositions are found in [4] .M Tarauceanu contributed to the study of Poset of Subhypergroup of a hyper group .He had drawn conclusions on Poset (Sub(H),⊆)and he had also given some open problems on above stated Poset and lattices . In this section, we study the distributivity of Poset of Subhypergroup of a Hyper group Principle Filter and we study the necessary condition for Poset to be a Distributive Poset Using the Concept of Translation,[7]. Finally we study the Annihilators [6] and ideals of Poset of Subhypergroup using Principle Filter. Notations and definitions are used from [4],[2],[6].

1.Basic Notations and Terminology:

Definition 1.1 [4]: A hyper Operation on H is a map o : H x H → P*(H).

Definition 1.2[4]: If the hyper operation" o "is associative and a o H=H=H o a, then (H, o) is a Hyper group.

Definition 1.3 [4]: A hyper group (H, o) is called a join space if "o" is commutative and a/b ≈ c/d ⇒ a od ≈ b o c.where a/b ={x ∈ H/a ∈ b o x}

2. Sub(L)=F(L)

Example 2.1[4] : Let (L; л;v) be a complete lattice and, for every a 2 L, denote by F(a) the principal Filter of L generated by a (F(L)={x ϵ L/a ≤ x})Then L is a join space under the hyper operation

a ◦ b = F(a^b); for all a; b ϵ L.

Proposition 2.2[4]: For the join space(L, ◦) given by the following equality holds Sub(L)=F(L)={F(a)/a ϵL} That is the subgroup of L coincides with the principal filter of L. In particular Sub(L) is a lattice anti isomorphic to L.

Definition 2.3: Let Sub(L)=F(L) be a Poset. If F(A) Sub(L)=F(L) then we will denote
L(F(A))={f(x) ϵ Sub(L),f(x)≤f(a) for all f(a) ϵ F(A)}

U(F(A))= {f(y) ϵ Sub(L),f(a)≤f(y) for all f(a) ϵ F(A) }

For all F(A),F(B),F(C) ϵ F(L)=Sub(L) Then

L.H.S= L (U (F (a), F (b)), F (c)) = L (F (a л b), F (c)) = (F ((a ∧ b) v c)

R.H.S=L (U (L (F (a),F(c)),L(F(b),F(c))))= L (U(F(a v c),F(b v c)))

=L (F ((a v c) ∧ (b v c)))

L(U(F(a),F(b)),F(c))= L(U(L(F(a),F(c)),L(F(b),F(c)))

The Poset Sub (L) =F(L) is distributive.

Definition 2.4: Let Sub(L)= F(L) be an Partially ordered set. The Subset I ⊆F(L) is called an ideal if F(a),F(b) ∈ I implies LU(F(a),F(b)) ⊆ I. Similarly we can define filter.

Illustration 2.5: Let L= {a, b, c, 1} and defined ∧ and ∨ by the following Cayley table

∨	0	A	b	1
0	{0}	{a}	{b}	{1}
a	{a}	{0,a}	{1}	{b,1}
b	{b}	{1}	{0,b}	{a,1}
1	{1}	{b,1}	{a,1}	L

∧	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Let I= {a, b, 1}

LU({a,1})=L({b,1})={a,1} ϵ I

LU({b,1})=L({a,1})=a ϵ I

LU({a,b})=L{1}=1 ϵ I

Clearly, I is an ideal.

Let F= {0, a, b}

U L ({a, b}) =U (0) =0

U L ({0, b}) =U (0) =0

U L ({0, a}) =U (0) =0

Clearly F is Filter.

Using, F(L)={x ϵ L/a ≤ x} and F(a) ϵ F(L)={F(a)/a ϵ L}

Let F(0),F(a),F(b) ∈ F(L)

Then, L.H.S=L(U(F(0),F(a)),F(b))=L(F(0∧ a),F(b)) = L(F(0),F(b))=L(F(0 ∨ b))=F(b).

Now

Consider,R.H.S=L(U(L(F(0),F(b)),L(F(a),F(b)))=L(U(F(∨b), F(a∨b))

=L(U(F(b) ∨F(1)))

=L(F(b∧ 1)=L(F(b))=F(b).

Therefore L.H.S=R.H.S

3. Sub (L)=L(G)

Example 3.1[4]: Let G be a Group. For all a,b ϵ G ,

We define a o b=<a, b>,the subgroup of G generated by a and b.Then $(G,_o)$ is a Commutative hyper group.

Result 3.2[4]: For the commutative hyper group G given by example the following Equality

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holds Sub (G)=L(G)That is the Subhypergroup of G coincide with the subgroups of G.

in Particular Sub(G) is a lattice.

Definition 3.3[2]: We assume that $G_a = \{g \in G /s(g) \leq a\}$It is easy that G_a is non empty .

Let $L(G)=\{ G_a /a \in P(X)$ and $G_{a \neq} ǿ\}$ and

we assume that $G_a \vee G_b=(G_x /a \vee b \leq x\}$ and

$G_{avb}= G_a \vee G_b=U(G_a , G_b)$ $G_{a\wedge b} = G_a \wedge G_b = L(G_a , G_b)$ $L(U((G_a, G_b), G_c) =L(G_a \vee G_b , G_c = (G_a \vee G_b) \wedge G_c$

Let $x \in L(U((G_a, G_b), G_c)$ Let $x \in (G_a \vee G_b) \wedge G_c$

Then $x \in (G_a \vee G_b)$ and $x \in G_c$

This Implies $a \vee b \leq x$ and $c \leq x$

That is $(a \vee b) \wedge c \leq x$

$(a \wedge c) \vee (b \wedge c) \leq x$

$a \wedge c \leq x$ or $b \wedge c \leq x$

$x \in G_a \wedge G_b$ or $x \in G_b \wedge G_c$

$x \in (G_a \wedge G_c) \vee (G_b \wedge G_c)$

This Implies $x \in U(L(G_a , G_c) ,L (G_b , G_c))$

Similarly we can Prove $x \in U(L(G_a , G_c) ,L (G_b , G_c))$

implies $x \in L(U((G_a, G_b), G_c)$

Sub (L)=L(G) is Distributive.

4.Translations

Definition 4.1 :-If F (L) is a Poset, then a mapping ϕ: F(L) → F(L) is called a lower Homomorphism of the Poset F(L) if ∀ F(a),F(b) ∈ F(L)

$U(\phi(L(F(a),F(b))) = U(\phi(F(a \vee b))) = U(F(a \vee b))$ and

$U(L(\phi(F(a)), \phi(F(b))) =U(L(F(a),F(b))) = U(F(a \vee b))$

$U(\phi(L(F(a),F(b))) = U(L(\phi(F(a)), \phi(F(b)))$

Theorem 4.2:- Poset is distributive if each it's translation is a lower hommorphism.

Proof:-Let the translation of F (L) is a lower homomorphism.

i.e., $U(\phi(L(F(a),F(b)))=U(L(\phi(F(a), \phi(F(b)))$

ϕ : F(L)→ F(L) defined by $\phi(F(a)) = F(a)$

L.H.S $= \phi (U (F(a),F(b))) = \phi(F(a \wedge b) = F(a \wedge b)$

R.H.S $= U (\phi (F (a)),F(b)) = U(F(a),F(b)) = F(a \wedge b)$

To prove that, Poset is Distributive

$L(U(F(a),F(b)),F(c))=L(U(L(F(a),F(c)),L(F(b),F(c))))$

L.H.S= $\phi(L(U(F(a),F(b)),F(c)) = \phi(L(F(a \wedge b),F(c)$

$= \phi(L(F(a \wedge b) \vee c))$

$= L \phi (F((a \wedge b)\vee c))$

$=L (F ((a \wedge b) \vee c))$

R.H.S=$\phi(L(U(L(F(a),F(c)),L(F(b),F(c))))= \phi(L(U(F(a \vee c),F(b \vee c))))$

$= \phi (L (F (a \vee c) \wedge (b \vee c))$

$=L (\phi (F (a \vee c) \wedge (b \vee c))))$

$= L (F (a \vee c) \wedge (b \vee c)))$

Now consider $F ((a \wedge b) \vee c)$

Let $x \in F ((a \wedge b) \vee c)$

$\leftrightarrow (a \wedge b) \vee c) \leq x$

$\leftrightarrow a \wedge b \leq x$ or $c \leq x$

$\leftrightarrow$ (a $\leq$ x and b $\leq$ x) or c $\leq$ x

$\leftrightarrow$ (a $\leq$ x or c $\leq$ x) and (b $\leq$ x or c $\leq$ x)

$\leftrightarrow$ (a v c $\leq$ x) and (b v c $\leq$ x)

$\leftrightarrow$ (a v c) $\wedge$ (b v c) $\leq$ x

$\leftrightarrow$ x $\in$ F((a v c) $\wedge$ (b v c))

F ((a $\wedge$ b) v c) = F((a v c) $\wedge$ (b v c))

Therefore, ϕ(L(U(F(a),F(b)),F(c))=
ϕ(L(U(L(F(a),F(c)),L(F(b),F(c))))

This Implies L(U(F(a),F(b)),F(c))=
L(U(L(F(a),F(c)),L(F(b),F(c)))

F (L) is distributive.

Theorem 4.3: If ϕ is Translation on a Poset F(L),F(x) $\in$ F(L), $\emptyset \neq$ B $\subseteq$ F(L) Then

ϕ(U(F(x),B))=U(f(x), ϕ(B))

Proof: Let F (z) $\in$ ϕ (U (F(x), B)

That is F (z) = ϕ (F(z1)),where F(z1) $\geq$ F(x) and F(z1) $\geq$ F(b) , F(b) $\in$ B

Let F (b) $\in$ B, F(z1) $\in$ U(F(x), F(b)),F(Z1) $\in$ ϕ(U(F(x),F(b))=U(F(x), ϕ(F(b))

But this Means that F(z) $\in$ U(F(x), ϕ(B))

Thus ϕ (U (F(x), B) $\subseteq$ U (F(x), ϕ (B))

Let F(w) $\in$ U(F(x), ϕ(B)) That is F(w) $\geq$ F(x) and F(w) $\geq$ ϕ (F(b)) for Each F (b) $\in$ B Then F(w) $\in$ U(F(x), ϕ (F(b)).Hence F(w)= ϕ (F(w1)) where F(w1) $\geq$ F(x) and F(w1) $\geq$ F(b).Thus , F(w1) $\in$ U(F(x),B)That is F(w) $\in$ ϕ(U(F(x),B).

Therefore U (f(x), ϕ (B)) $\subseteq$ ϕ (U (F(x), B))

Similarly we can Prove that ϕ (U (F(x), B)) $\subseteq$ U (f(x), ϕ (B))

5.Annihilators

Definition 5.1: Let Sub(L)= F(L) be an Partially ordered set. The Subset I $\subseteq$ F(L) is called an ideal if F(a),F(b) $\in$ I implies LU(F(a),F(b)) $\subseteq$ I

Definition 5.2: Let Sub(L)=F(L) be a Poset F(A) $\subseteq$ F(L), F(B) $\subseteq$ F(L) ,then annihilator in the set is defined by

$<$ F(A),F(B) $>$ = {F(x) $\in$ F(L);UL(F(A),F(x)) $\supseteq$ U(F(B))}

Theorem 5.3:- An Poset sub(L) is distributive if and only if each annihilator in sub(L) is an ideal in sub(L)

Proof:-i) let sub(L) be a distributive set, and <F(a),F(B)> be an annihilator in S.

let x,y $\in$ <F(a),F(B)>.Then

U L(F(a),F(x)) $\supseteq$ U(B)

U L(F(a),F(y)) $\supseteq$ U(B)

Let F(Z) $\in$ L U(F(x),F(y)) Then

L(F(Z)) $\subseteq$ L U(F(x),F(y)),

U(F(Z)) $\supseteq$ U(F(x),F(y))

U L(F(a),F(Z)) = U L(F(a),U(F(Z))) $\supseteq$ U L(F(a),U(F(x),F(y))).

By the distributive law, the right side of the last inclusion is equal to

U L U (L (F(a),F(x)),L(F(a),F(y))

= U(L(F(a),F(x)),L(F(a),F(y)))

=U L(F(a),F(x)) $\cap$ U L(F(a),F(y)) $\supseteq$ U(B).

Hence U L (F(a),F(Z)) $\supseteq$ U F(B) and Z ϵ <F(A),F(B)>

Thus L U(F(x),F(y)) $\subseteq$ <F(a),F(B)> and <F(a),F(B)> is an ideal.

ii) Let every annihilator in S is an ideal.

F(a),F(b),F(x) ϵ $\mathrm{sub}(L) = F(L)$

Then,UL(F(a),F(x)) $\supseteq$ U $L(F(a),F(x)) \wedge$ U $L(F(b),F(x))$ and analogously

UL(F(b),F(x)) $\supseteq$ U $\left(L(F(a),F(x)),L(F(b),F(x))\right)$

Hence for F(B) = L(F(a),F(x)) $\cup$ $L(F(b),F(x))$

it holds, F(a) ϵ $< F(x),F(B) >$

F(b) ϵ $< F(x),F(B) >$

But <F(x),F(B)> is an ideal.

L $\cup$ (F(a),F(b)) $\subseteq$ $< F(x),F(B) >$ (1)

LetF(Z) $\in L\left(U(F(a),F(b)),F(x)\right)$, then F(Z) L $\cup(F(a),F(b)) \cap L(F(x))$and By (1)

F(Z) ϵ $< F(x),F(B) >$

Therefore

$\cup L(F(Z),F(x)) \supseteq \cup\left(L(F(a),F(x)),L(F(b),F(x))\right)$

Moreover,x $\in L(F(x))$ implies $L(F(Z),F(x)) = L(F(Z))$,

Thus we obtain,

$\cup(F(Z)) \supseteq \cup\left(L(F(a),F(x)),L(F(b),F(x))\right)$,

L(F(Z)) $\subseteq$ L $\cup$ $(L(F(a),F(x)),L(F(b),F(x))$

i.e.,L($\cup$ $(F(a),F(b)),F(x)$)
$\subseteq$ L $\cup$ $(L(F(a),F(x),L(F(b),F(x))$

Similarly we can prove the converse inclusion proving distributivity of S.

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