A New Approach for Approximate Modeling and Controller Design of SISO Multiple Time Delay System

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Abstract—This paper presents a new approach for approximation of multiple time delay transfer function model to a single input/output delay transfer function. The methodology uses concept of model matching technique of comparing an approximated generalized time moments (AGTM)/ approximated generalized markov parameters (AGMP) of one system representation to those of approximated models. A classical PI/PID Controller is designed for multiple time delay systems (MTDS) using the same methodology. Genetic algorithm is used for further fine tuning AGTM/AGMP matching method. Numerical examples are presented to demonstrate the effectiveness of the proposed method.

Keywords- *Multiple time delay systems, Model matching concept,AGTM/ AGMP matching method, , Genetic algorithm.*

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I. Introduction

Time-Delay Systems (TDS) arise in many applications from diverse areas such as economy, biology, process systems, traffic flow and communication systems. Delays can appear due to various factors such as transport phenomena, computation of the control input, time-consuming information processing in measurement devices, etc. Time delay problems are often solved indirectly by using approximation. A widely used approximation method is the Pade approximation, which is a rational approximation and results in a shortened fraction as a substitute for the exponential time-delay term in the characteristic equation [1-2]. However, such an approach constitutes a limitation in accuracy, can lead to instability of the actual system and induce non-minimum phase and, thus, high-gain problems [3]. Prediction-based methods (such as Smith predictor [4], finite spectrum assignment (FSA)) [5], and adaptive Posicast [6] have been used to stabilize time-delay systems by transforming the problem into a non-delay system. Such methods require model-based calculations, which may cause unexpected errors when applied to a real system. Furthermore, safe implementation of such methods is still an open problem due to computational issues. Controllers have also been designed using the Lyapunov framework (e.g., linear matrix inequalities (LMIs) or algebraic Riccati equations (AREs)) [7-8]. These methods require complex formulations, and can lead to conservative results and possibly redundant control.

In the past decades, model approximation problem have received considerable attention. In the literature some important results reported such as Pade approximation, Routh approximation and many of their variants [1,2,9-12]. Most real processes cannot be accurately modeled without introducing delays. A high order model can be effectively represented by a low order one with a time delay. By introducing time delay to approximate model substantially the approximation might be improved [10,11]. Model reduction with time delay, response matching technique and genetic algorithms are developed in the literature [9-13].

In the case of Multiple Time Delay (MTD) system, the delay term is not only present in numerator but also in denominator of the transfer function [14-15]. This denominator delay increases the complexity of the system. Reasons for Multiple Time Delay are the complexities in the system's physical structure and its dynamics.

The mathematical models for many process control systems can be represented by a specific multiple time delay continuous time transfer function with a delay free denominator and a multiple time-delay numerator. Again it can be represented as a general multiple time delay continuous time transfer function with multiple time delays in both the denominator and numerator.

Attempts have been made by researchers in past for development of a single delay model representation of MTD system. In [16], a method was proposed to convert MTD system representation into a transfer function model with single input/output delay by applying approximation methods, Hankel matrices, singular value decomposition, modified z-transform method [17] and minimal realization methods [18,19].

The proposed method in this paper works on the same objective as in [16] but it is achieved by applying a new simple methodology. The methodology uses the concept of Model Matching technique and AGTM/ AGMP matching method [20].

The paper is organized as follows. Section 2, deal with the problem formulation. In section 3, two methodologies are proposed to convert the MTDS transfer function into a transfer function representation with a single time delay. Numerical examples are also given in this section to show the effectiveness of the proposed methods. Results are compared with that obtained in [16]. In section 4, controller is designed for MTD system to achieve a desired response. Finally Section 5 concludes the paper.

II. Problem Formulation

Generally MTD systems are represented in state space form. A general Single Input Single Output MTD system having delays in states x(t), inputs u(t) and outputs y(t) is represented as in (1)

$$\dot{x}(t) = A\,x(t-\delta) + B\,u(t-\gamma)$$
$$y(t) = C\,x(t-\varsigma) \tag{1}$$

where $\delta \geq 0$ is delay in states, $\gamma \geq 0$ is delay in input and $\varsigma \geq 0$ is delay in output. A, B and C are constant system matrices with appropriate dimensions.

A single input single output MTD system in transfer function form can be represented as Eqn. (2).

$$G(s,e^{-Ts}) = \frac{N(s,e^{-T_n s})}{D(s,e^{-T_d s})} = \frac{\sum_{i=0}^{m} b_i\, s^i\, e^{-T_{ni} s}}{\sum_{j=0}^{n} a_j\, s^j\, e^{-T_{dj} s}} \tag{2}$$

where $N(s,e^{-T_n s})$ and $D(s,e^{-T_d s})$ are numerator and denominator of the MTD system transfer function with delay time 'T_n' and 'T_d' respectively. T_{ni} and T_{dj} where i=1 to m, j=1 to n are delay time associated corresponding to coefficients of the numerator and denominator.

A general transfer function model with n poles, m zeros and time delay 'τ' can be represented as,

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} e^{-\tau s} \quad ; n \geq m \tag{3}$$

The objective is to approximate multiple time delay transfer function in Eqn. (2) to the transfer function model with single input/output delay as in Eqn. (3).Thus

$$G(s,e^{-Ts})\Big|_{s=\delta_i} = G(s)\Big|_{s=\delta_i} \tag{4}$$

Where i=1,2,……,(n+m+1+1) and δ_i is real or complex points in s-plane.

III. Modeling Methodologies

To convert MTDS into single delay transfer function two methods are proposed here. Method I yields a non linear equations where as method II results in linear equations, solution of which gives desired unknown coefficients in the approximated model.

A. *Proposed Method I*

Generate the unit-step response data of the MTDS transfer function. Select the model order to approximate the MTDS transfer function.

The problem is to find the unknown coefficients and time delay 'τ' of approximate model transfer function in Eqn. (4). AGTM/AGMP method is employed to get these unknowns coefficients.

AGTM method [20] is matching the frequency response of actual and approximated models at different points in the s-plane. These points are expansion points (nonzero real values). The number of expansion points is same as the number of unknowns in the Eqn. (4). Equate the response of actual and approximate transfer function at expansion points. Substitute the expansion points in Eqn. (4) and get the equations with unknown parameters. Here, in this case unknown is in the exponent so equations are nonlinear. Solve these equations with an initial vector 'x_0'.

$$x_0 = [b_{m,0}\, b_{m-1,0} \dots b_{1,0}\, b_{0,0}\, a_{n,0}\, a_{n-1,0} \dots a_{1,0}\, a_{0,0}\, \tau_{d,0}] \tag{5}$$

The solution of equations are unknown coefficients of approximate model transfer function and time delay. By using this solution form the approximated model with time delay. Get the step response data of this approximated model. The matching effectiveness of the approximate model is based on the performance index value 'J'.

$$J = \int_0^\infty \left(y_a(t) - y_m(t)\right)^2 dt \tag{6}$$

where $y_a(t)$ is actual MTDS step response and $y_m(t)$ is approximated model step response. Search for minimum 'J' by varying expansion points, initial vector or both and select the corresponding model as the best approximated model.

Example:

To show the effectiveness of proposed approximated modeling methodology, let us consider a general multiple time delay transfer function with a long state delay in the denominator given by [8].

$$G(s,e^{-Ts}) = \frac{(s+2)e^{-3s} + (s+3)e^{-2s}}{(6s+1) + (0.5s+1)e^{-5s}} \tag{7}$$

Direct simulation of the MTDS transfer function in Eqn. (7) is not possible. Hence an alternate representation of the transfer function in Eqn. (7) is shown in Fig.1. It is noted that a sensor with dead time of 5 sec is in the output feedback loop, this delay would become as internal state delay in state space representation. This term is going to appear in the denominator of the transfer function.

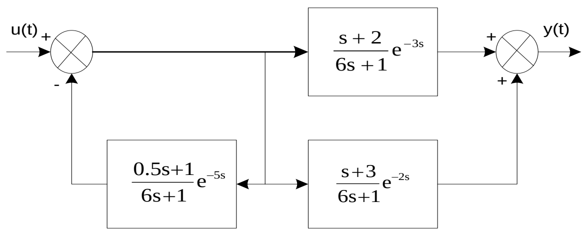

Fig.1. Block diagram for unit step response generation

The MTD system transfer function in Eqn. (7) is approximated to a second order denominator and first order numerator with a single input/output delay by applying method I. The minimum value of the performance index 'J' is 0.0251 with expansion point vector [0.1 0.5 0.8 1 1.2 0.3] and the corresponding initial vector [-12.0075 -12.0075 -12.0075 -12.0075 -12.0075 -12.0075]. The approximated transfer function model obtained is,

$$\frac{Y(s)}{U(s)} = \frac{5.531s + 3.349}{9.649s^2 + 3.319s + 1.345} e^{-1.84\,s} \qquad (8)$$

B. Limitations of method I

Here the equations are nonlinear and the solution depends on initial values. Also complexity will increase with increase in range and magnitude of expansion points, initial vectors or both. These limitations are overcome in method II.

C. Proposed Method II

This method matches the frequency response of the model on jω axis of s-plane [21]. Compare the magnitude of Eqn. (4).

$$\left|G(s, e^{-Ts})\right| = \left|\frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}\right| \qquad (9)$$

since $\left|e^{-\tau s}\right| = 1$

Rearranging Eqn. (4) one may yield,

$$e^{\tau s} = \frac{\left(\dfrac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}\right)}{G(s, e^{-Ts})} \qquad (10)$$

Equate the phase angle on both sides of Eqn. (10).

$$\tau = \angle\left(\frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}\right) - \angle G(s, e^{-Ts}) \qquad (11)$$

By AGTM/AGMP method calculate the unknown coefficients in Eqn. (9). Make use these calculated unknown coefficients to find the delay 'τ' in Eqn. (11) at the same expansion points. Form the approximate model with these calculated unknowns. Use the performance index in Eqn. (6) to judge the effectiveness of the approximation. One can use genetic algorithm to find the optimum expansion points.

But the method in [21] matches the poles of the approximated model at the same points (repeated poles at same point). This is not valid for all the cases. To avoid this, the approximate model is changed to Eqn. (12).

$$G(s) = \frac{(s+z)(s+z+d_1)\ldots(s+z+d_{m-1})}{(s+p)(s+p+c_1)\ldots(s+p+c_{n-1})} e^{-\tau s} \quad ; n > m \qquad (12)$$

The first pole of Eqn. (12) is at 'p' and the remaining poles are c_1, c_2… c_{n-1} distances away from the first pole in the opposite direction to origin. Similarly the first zero of Eqn. (12) is at 'z' and remaining zeros are d_1, d_2… d_{m-1} distances away from the first zero in the opposite direction to origin in s-plane.

Compare the magnitude and the phase on Eqn. (12). Use AGTM/AGMP method, form the approximate model and search for minimum performance index.

As an example consider MTDS transfer function in Eqn. (7) which is approximated to three poles and two zeros transfer function with input/output delay using method II. The minimum performance index is 0.2081 with expansion points [0.0059 0.2765 -1.0071 -1.9401 -0.6866 -0.9092 -0.9490 1.0972 -0.2772] and the approximated transfer function model is in Eqn. (13).

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 2.959s + 1.545}{s^3 + 4.228s^2 + 1.45s + 0.6219} e^{-2.07\,s} \qquad (13)$$

D. Results Comparisons

Unit step response comparisons of actual MTDS, approximated model by method I and method II is shown in Fig.2.

Comparisons of results with results by L.B.Xie [16] for time 0 to 70 sec is in Table 1. From the Table 1 the proposed method I show best matching among all the methods for the considered example and the approximated model order was reduced from 6 to 2.

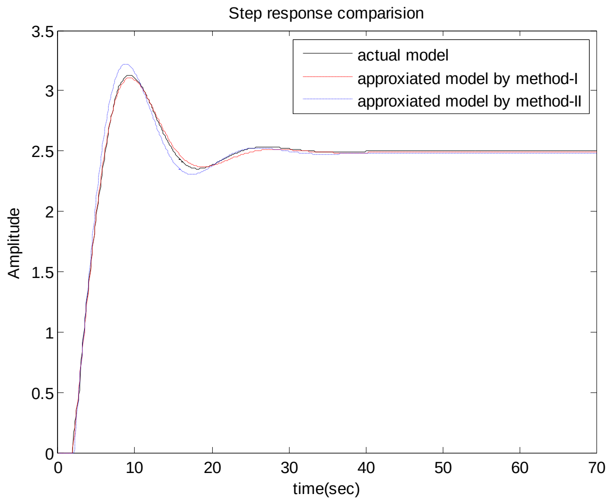

Fig.2. Unit step response comparisons

TABLE I. COMPARISON OF RESULTS

Parameter	L.B.Xie [16]	Proposed Method I	Proposed Method II
Performance index	0.069	0.0251	0.2081

IV. CONTROLLER DESIGN

In this section a conventional PI or PID or LEAD/LAG controller is designed for MTDS. The control objective is to design a controller to MTDS to achieve desired response.

The design methodology is a combination of model matching and AGTM/AGMP method. The control scheme is shown as block diagram in Fig.3.

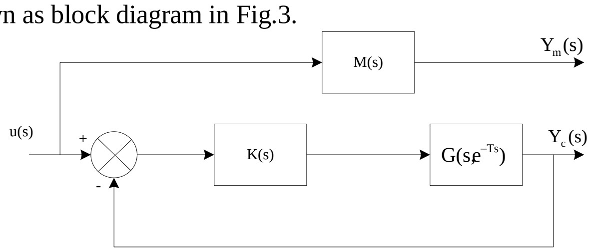

Fig.3. Block diagram of desired model and designed closed loop system

Here M(s) is the model to be achieved with desired specifications (time/frequency domain), K(s) is higher order controller and $G(s, e^{-Ts})$ is MTDF transfer function.

By model matching method,

$$y_m(s) = y_c(s) \qquad (14)$$

Thus

$$M(s) = \frac{G(s, e^{Ts}) K(s)}{(1 + G(s, e^{Ts})) K(s)} \quad (15)$$

From Eqn. (15),

$$K(s) = \frac{M(s)}{[1 - M(s)] G(s, e^{Ts})} \quad (16)$$

K(s) is again a multiple time delay transfer function. The general forms of the conventional controllers are given in Eqn.s (17) to (19).

PID controller: $$C(s) = k_P + \frac{k_I}{s} + k_D s \quad (17)$$

PI controller: $$C(s) = k_P + \frac{k_I}{s} \quad (18)$$

Lead/Lag controller: $$C(s) = \frac{k(s + a)}{(s + b)} \quad (19)$$

The higher order controller K(s) is approximated to a conventional controller given in Eqn. (17) to (19) by using AGTM/AGMP method. Select the expansion points and should be equal to number of unknowns in the selected controller form. For example a PID controller has three unknowns in its form. They are K_P, K_I and K_D. Equate K(s) to C(s) at expansion points.

$$C(s)\big|_{s=\delta_i} = K(s)\big|_{s=\delta_i}, i = 1 \text{ to } n_e \quad (20)$$

Where 'n_e' is number of unknown parameters in and 'δ_i' represents i^{th} the expansion point. Get the equations at expansion points and find the unknowns in Eqn. (20). Form the controller with these calculated values. The closed loop system with controller is in Eqn. (21).

$$C_L(s) = \frac{G(s, e^{-Ts}) C(s)}{1 + G(s, e^{-Ts}) C(s)} \quad (21)$$

Calculate the unit step response data of the model M(s) with desired specifications and also unit step response data of the formed closed loop system in Eqn. (21). Calculate the performance index J by Eqn. (22).

$$J = \int_0^{\infty} (y_m(t) - y_c(t))^2 \, dt \quad (22)$$

where $y_m(t)$ is unit step response data of desired model M(s) and $y_c(t)$ is unit step response data of closed loop system with controller . Check for minimum value of performance index. Use genetic algorithm to find the optimum expansion points.

Example 1

To show the effectiveness of this method, a PI controller is designed for the MTDS transfer function in Eqn. (7) to achieve a model with specifications, time delay 2.5 sec, undamped natural frequency of 1 rad/sec and damping factor of 4. The model with these specifications is in Eqn. (23).

$$M(s) = \frac{1}{s^2 + 8s + 1} e^{-2.5s} \quad (23)$$

For this case using model matching methodology the higher order controller K(s) is,

$$K(s) = \frac{\{(6s+1) + (0.5s+1)e^{-5s}\} \; e^{-2.5s}}{\{s^2 + 8s + 1 - e^{-2.5s}\} \{(s+2)e^{-3s} + (s+3)e^{-2s}\}} \quad (24)$$

For PI controller, optimum expansion points 0.9621, 0.1406 and K_P=0.0486, K_I=0.0363. Unit step response comparisons of model with desired specifications and the closed loop controller are in Fig.4.

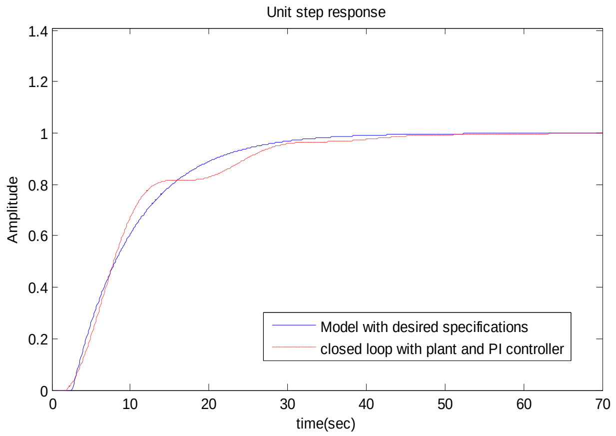

Fig.4. Unit step response comparisons (for example 1)

Example 2

For same MTDS transfer function in Eqn. (7) a PID controller is designed to achieve a model with specifications time delay 2 sec, undamped natural frequency of 0.5 rad/sec and damping factor 7. The model with these specifications is in Eqn. (25).

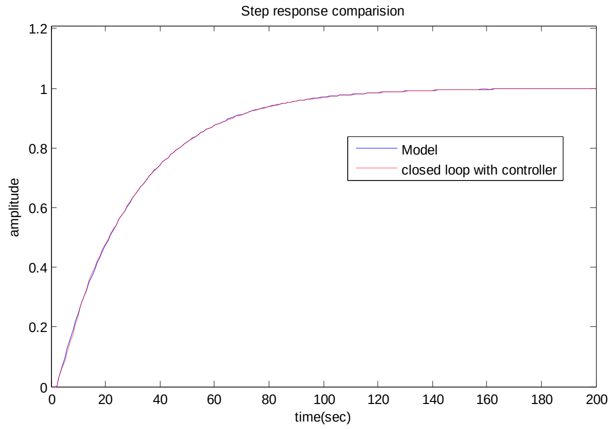

Fig.5. Unit step response comparisons (for example 2)

$$M(s) = \frac{0.25}{s^2 + 7s + 0.25} e^{-2s} \quad (25)$$

The higher order controller K(s) for this case is,

$$K(s) = \frac{\{(6s+1) + (0.5s+1)e^{-5s}\} \; e^{-2s}}{(s^2 + 7s + 0.25(1 - e^{-2s})) \{(s+2)e^{-3s} + (s+3)e^{-2s}\}} \quad (26)$$

For PID controller, optimum expansion points 0.0129, 0.1338, 0.07 and K_P=0.0101, K_I=0.0133, K_D=0.0467. Unit step response comparisons of model with desired specifications and the closed loop controller are in Fig.5.

V. Conclusions

This paper presented a new approach for approximation of multiple time delay system transfer function into a single transfer function with only single input/output delay. The proposed methods are simpler and shows best matching compared to the other methods in Literature. The order of approximation is also reduced with these methods. Controller is designed for MTDS transfer function by AGMT/AGMP method to achieve the desired response from the system.

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REFERENCES

[1] Pal.J "Improved Pade approximant using stability Eqn. methods", Eectron. Lett., vol. 19, no. 11, pp.436-427, July 1983.

[2] Guillermo J. Silva, Aniruddha Datta and S.P. Bhattacharyya, 'Controller Design via Pade Approximation Can Lead to Inst ability", IEEE Conference and Decision and Control Orlando, Florida USA, Deewber 2001(4733-4737).

[3] Silva,Datta,Battacharyya,"PID controllers for Time Delay Systems", Birkhauser,Boston,2005.

[4] Smith, O.J.M. "Closer control of loops with dead time", Chemical Engineering Progress, 53,217-219. (1957).

[5] Qing-Chang Zhong, "Robust Control of Time-delay Systems", Springer-Verlag London Limited 2006.

[6] Niculescu, S.-I., & Annaswamy, A. M. (2003). "An adaptive smith-controller for time-delay systems with relative degree n^2", Systems & Control Letters, 49,347_358.

[7] Gu. K., & Niculescu, S.-I. (2003), "Survey on recent results in the stability and control of time-delay systems", Transactions of ASME, 125, 158_165.

[8] Boukas. E. K. and Z.-K. Liu (2002), "Deterministic and tochastic time delay systems", Birkhauser, Boston.

[9] Y. Liu and B. D. O. Anderson, "Model reduction with time delay," IEEE Proc. D, vol. 134, no. 6, pp. 349-367, 1987.

[10] Z. J. Yang, T. Hachino, and T. Tsuji, "Model reduction with time delay combining the least squares method with the genetic algorithm", IEEE Proc. of Control Theory and Applications, vol. 143, no. 3, pp. 247-254, 1996.

[11] S. Mukherjee, Satakshi, and R.C. Mittal, "Model Order reduction using response-matching technique," J. Franklin Inst., vol. 342, no. 1, pp. 503-519, 2005.

[12] Ping Zhou, Bo Xiang, Jun Fu and Tian-You Chai,"Model approximation of multiple delay transfer function models using multiple-point step response fitting",International Journal of Control, Automation, and Syatem (2012) 180-185.

[13] H. X. Li, J. S. Lu, and H. S. Yang, "Model reduction using the genetic algorithm and Routh approximation," J. Syst. Eng. and Electro., vol. 16 no. 3, pp. 632-639, 2005.

[14] N.F.Jerome and W.H. Ray, " High-performance multivariable control strategies for systems having time delays",AIChE Journal 25 (1979) 1043-1056.)

[15] T. Kaczorek, "Realization problem for a class of positive continuous-time systems with delays", International Journal of Applied Mathematics and Computer Science 15 (2005) 447–453.

[16] L.B. Xie,L.S. Shieh,J.S.H.Tsai and Y. Zhang, "Approximated modeling and minimal realization of transfer function matrices with multiple time delays", Journal of Process Control 23 (2013) 3– 11.

[17] J.T. Tou, "Digital and Sampled-Data Control Systems", McGraw-Hill, New York, 1959.

[18] M. Fliess, R. Marguez and H. Mounier, "An extension of predictive control, PID regulators and smith predictors to some linear delay systems", International Journal of Control 75 (2002) 728–743.

[19] H.P. Wang, L.S. Shieh, Y. Zhang and J.S.H. Tsai, "Minimal realization of the transfer function matrix with multiple time delays", IET Control Theory & Applications 1 (2007) 1294–1301.

[20] T.K. Sunil Kumar, "Model Matching Controller design methods with applications in Electrical Power Systems" Ph.D thesis, IIT Kharagpur 2009.

[21] Qing-Guo Wang, Qiang Bi and Yong Zhang, "Re-design of Smith predictor systems for performance enhancement" ISA Transactions 39 (2000) 79-92.