

# Deflection Characteristics of Shallow Funicular Concrete Shells over Rectangular Ground Plan Ratio 1:0.5

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**Abstract:**-Shells, stressed skin structures because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. Concrete shallow funicular shells of rectangular ground plan, double curvature with different rises are loaded to failure with a concentrated central force. Specimens of size 100 cm x 50 cm in plan with edge beam of size 4cm x 4cm are prepared with concrete of grade M30 for which the mix design is carried by Indian standard method. The specimens are prepared with various rises and moist cured. They are subjected to ultimate loads and the corresponding stains and deflections are measured. Failure patterns for shells with different rises are observed. From the experimental investigations a relation between span to rise ratio and ultimate load is arrived. It is concluded that the ultimate loads are function of the rise of the shell.

**Keywords**– Shells, Concrete shell, Funicular shell, Ultimate load, span to rise ratio

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## 1.0 INTRODUCTION

Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane (1). In the design of new forms of concrete shell structures the conventional practice is to select the geometry of shell first and then making the stress analysis. In this process no deliberated effort is taken to ensure the desirable state of stress in the material. Perhaps it is more logical to reverse this process. Ideally a concrete shell in its membrane state carries the external loads by pure compression, unaccompanied by shear stresses so that no tensile stresses develop and hence the reinforcement becomes necessary excepting for secondary effects like bending, shrinkage. In most of the shell roof is the predominant load is the dead weight. Hence it is advantageous to select the shape of shell in such a way that, under this condition of loading, the shell is subjected to pure compression without bending. This can be achieved by shaping the shell in the form of a catenary which the funicular shape is corresponding to the dead weight (2). Shell of rectangular and square ground plans are very frequent occurrence in practice. An attempt is made to study the influence of rise on the ultimate load of the Shallow Funicular Concrete Shells of rectangular Ground Plan ratio of 1:0.5

John W Weber et al., observed that the mathematical investigations of shallow funicular shells with large concentrated loads should be based on large deflection theory

and the deflection characteristics of a shell vary closely with its rise parameter (3). Patricia M Belles et al., conclude that the analysis of the stresses and deformations of concrete shell with the anti funicular shape found with the homeostatic model technique (HMT) allows the verification of quasi membrane behaviour (4). Vafai and Farshad studied that the experimental failure loads are found to be directly related to the amount of reinforcement and the age of concrete shells (5). Sachithanantham et al, concluded that the deflection of shallow funicular concrete shells decrease with increase in rise within elastic range and also concluded that the ultimate load carrying capacity increases with increase in rise (6).

## 2.0 METHODOLOGY

### 2.1 Materials

Concrete funicular shell specimens of various rises are prepared with cement, fine aggregate and coarse aggregate for which the design mix proportion is arrived as shown in table 2.2. To investigate the influence of different rises on the ultimate strength of shallow funicular shells, specimens are prepared and designated as follows.

- i) SFS I – Shallow Funicular Shell with rise (r1) – 5.1 cm
- ii) SFS II – Shallow Funicular Shell with rise (r2) – 6.5 cm
- iii) SFS III – Shallow Funicular Shell with rise (r3) – 8.7 cm

Preliminary tests are carried as per IS standards on the material used for concrete like specific gravity, fineness,

4873

consistency, and initial setting time for cement. For fine and coarse aggregates tests such as sieve analysis, specific gravity, impact value, crushing value and abrasion value (Los Angeles and Deval's) are conducted as per standards(7)(8) and the results are tabulated.

**Test Data for Materials**

Cement used	-	PPC – 53 grade
Specific gravity of Cement	-	3.15
Specific gravity of coarse aggregate	-	2.76
Specific gravity of Fine aggregate	-	2.65
Water absorption		
Coarse aggregate	-	0.3%
Fine aggregate	-	2.9%
Free surface moisture		
Coarse aggregate	-	Nil
Fine aggregate	-	2.6%
Sieve analysis		
Coarse aggregate	-	Confirms grading of IS 383 - 1973
Fine aggregate	-	Confirms zone - II

The design stipulations for M30 grade concrete is given in table 2.1.

**Table 2.1 Design Stipulations for M30 grade Concrete**

Design Stipulations	M30
Characteristic Compressive Strength	30 N/mm <sup>2</sup>
Maximum size of aggregates	10 mm (angular)
Degree of Workability	0.85 (Compacting Factor)
Type of Exposure	Mild
Degree of Quality Control	Very Good

**Table 2.2 Design Mix proportion**

Grade	Cement	Fine Aggregate	Coarse Aggregate	w/c ratio
M30	1	1.327	2.255	0.47

**2.3 Preliminary Investigations**

The following tests are conducted on cement, fine aggregate and coarse aggregate and the results are tabulated in table 2.3.

**2.2 Mix Design**

Concrete used for the investigation is designed in accordance with IS 10262 (9).

**Table 2.3 Test on Cement, Fine Aggregate and Coarse Aggregate**

Materials	Properties	Values
Cement	Specific Gravity	3.15
	Fineness, %	95.32
	Consistency, %	32
	Initial Setting time, min	38
Fine Aggregate	Specific Gravity	2.65
	Gradation	Zone II
Coarse Aggregate	Specific Gravity	2.76
	Impact Value, %	26.30
	Crushing Value, %	15
	Los Angeles Abrasion Value, %	8

**2.4 Casting of Shallow Funicular Pre moulds**



**Fig.2.1 Rectangular steel frame with flexible PU membrane for casting the pre mould**

### 2.5 Casting of Shallow Funicular Moulds using Pre moulds

The concrete pre moulds are inverted and kept on a flat surface. The funicular surface profile of the pre mould is lubricated with demoulding agent. Fresh concrete with good workability is poured over the pre mould in such a way that the convex profile of funicular shape of the pre mould makes the concave profile of funicular shape in the mould.

Concrete funicular moulds of size 100cm x 50cm in plan with provision of edge beam of 4cm x 4cm are prepared with adequate reinforcement. Fabrication of mould is in such a way that the moulding of four edge beams is provided as an integral component of each shell mould. By repeating this process shell moulds of various rises  $r_1, r_2$  and  $r_3$  are prepared as shown in Fig 2.2.

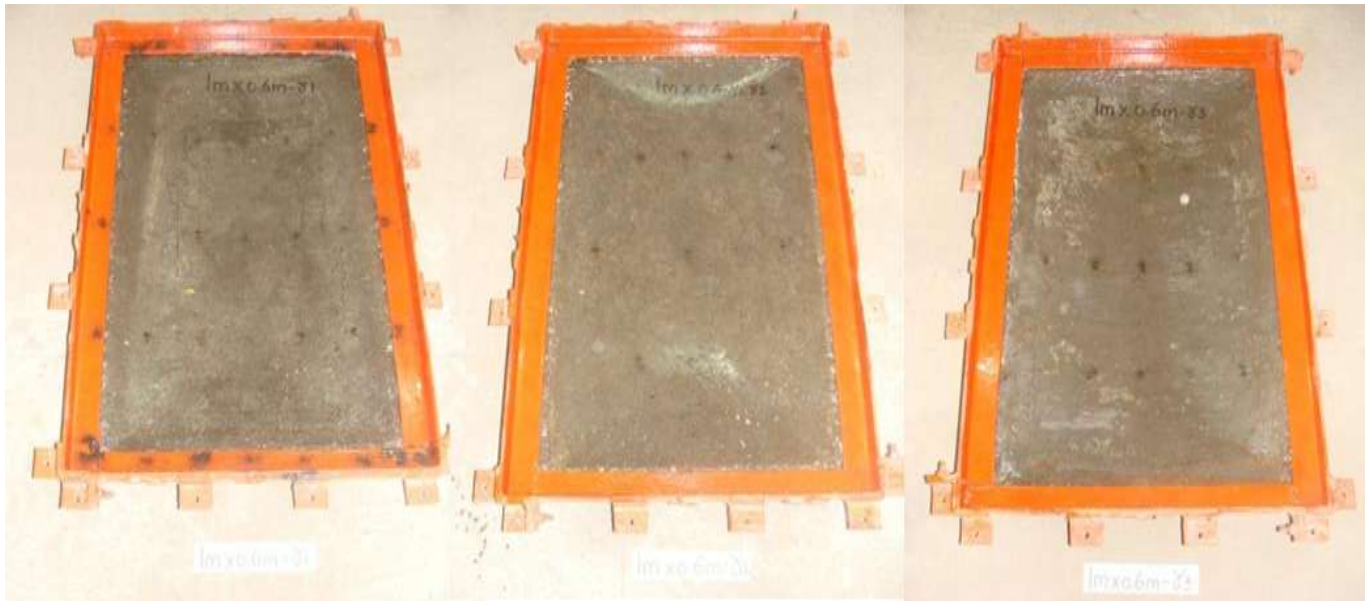


Fig.2.2 Moulds of Shallow Funicular Shells

### 2.6 Casting of Shallow Funicular Shells

Concrete funicular shells of size 100cm x 50cm in plan with edge beam of 4cm x 4cm are prepared using cement concrete of grade M30 with 4mm diameter GI wires at a spacing of 75mm c/c as reinforcement. Rectangular Shell specimens are prepared with shell moulds with various rises of 5.1 cm ( $r_1$ ), 6.5 cm ( $r_2$ ) and 8.7 cm ( $r_3$ ) as shown in Fig 2.4. Care is taken to maintain the uniform thickness of funicular shell as 25mm with the help of measuring gauge. The shell specimens are moist cured.



Fig.2.3 Casting of shell specimen



Fig.2.4 Shell specimens with various rises

### 3.0 EXPERIMENTAL SETUP AND TESTING

The self-straining load frame and the Hydraulic loading jack along with Load cell are arranged in such a way to apply the concentrated force over the centre of the shell specimen as shown in Fig. 3.1. Care is taken to avoid eccentricity during loading. Linear Variable Differential Transformer (LVDT) and electrical resistance strain gauges are mounted where the deflection and strain are required in the specimen. To facilitate the locations of LVDTs and strain gauges the specimens are specially painted and the surface of the shell is discretized with 200 elements of size of 50mm x 50mm. Specimens and the grids are marked as shown in Fig. 3.2. The rise of the shell specimens cast are measured using Total Station and it is observed that the rises are almost equal to the predetermined values. Shells of SFS I, SFS II and SFS III are placed on loading frame and subjected to central concentrated force and the corresponding deflections are measured within the elastic range using a 20 channel data acquisition system.



Fig. 3.1 Experimental Setup

After the elastic range all the specimens are subjected to failure and hence the ultimate loads are recorded in the data acquisition system. Visible cracks first appeared at the centre of the shell's outer surface and then propagated towards the corners along the diagonals. As the load is increased apparent zones of tension near and approximately parallel to the supports are also cracked by which the shell eventually failed. The crack patterns of shell specimens are shown in fig. 3.3.

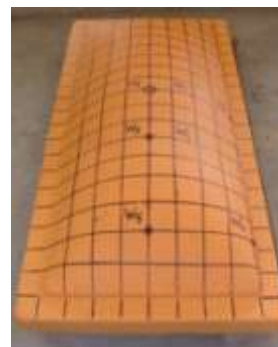


Fig. 3.2 Discretized Shell specimen



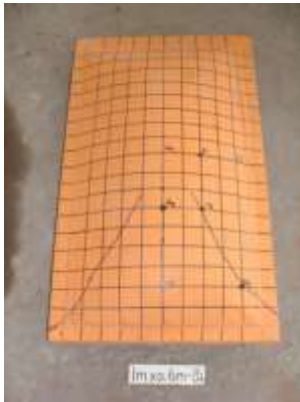
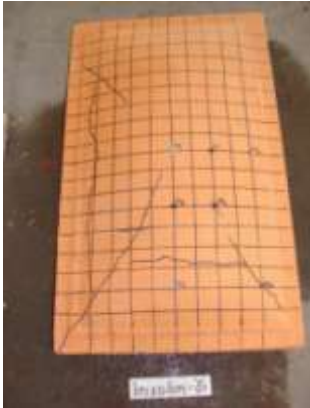


Fig. 3.3 Crack patterns of shell specimens

4.0 RESULTS AND DISCUSSIONS

From the experimental investigations of SFS I, SFS II and SFS III a plot is made between the load and the corresponding deflection as shown in fig. 4.1, 4.2 and 4.3 for rise r1, r2 and r3 respectively.

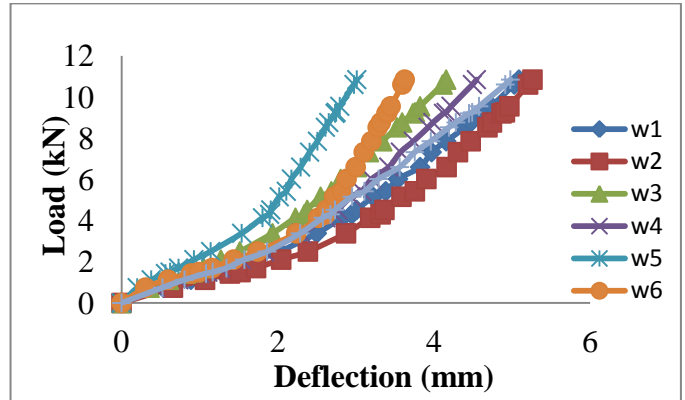


Fig.4.1 Load vs deflection, r1

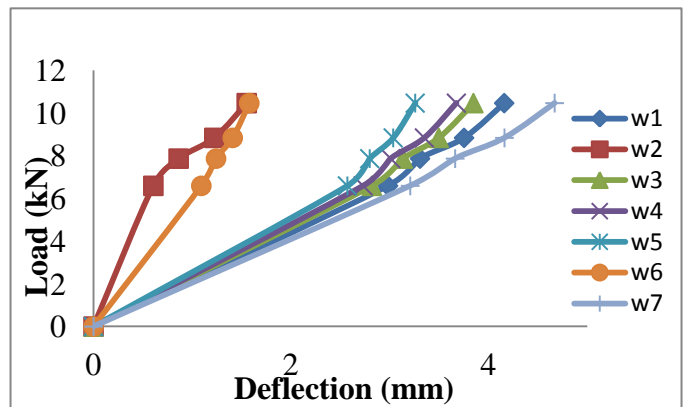


Fig.4.2 Load vs deflection, r2

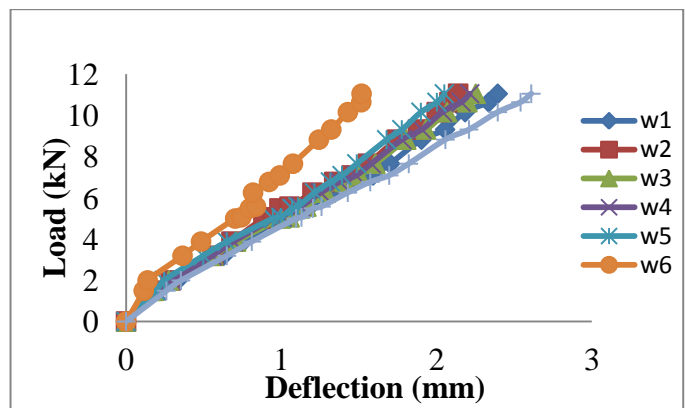


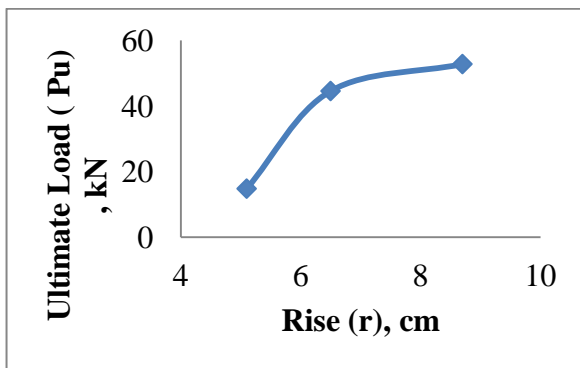
Fig.4.3 Load vs deflection, r3

From the figures 4.1, 4.2 and 4.3 it is observed that the deflection of shallow funicular concrete shell decreases with increase in rise. The ultimate loads for the specimens are tabulated in table 4.1.

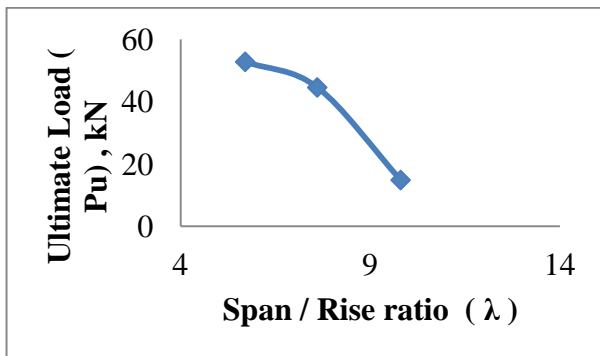
**Table 4.1: Test results of Ultimate load (Pu) for shells**

Type	Rise, (r)	Span/Rise ratio, ( $\lambda$ )	Ultimate Load, Pu (kN)
SFS I	5.1	9.8	14.84
SFS II	6.5	7.6	44.6
SFS III	8.7	5.7	52.82

A plot is made between ultimate load and the rise of the shell as shown in Fig 4.4. It is observed that the ultimate load increases with increase in rise. A plot is made between ultimate load and span to rise ratio ( $\lambda$ ) as shown in Fig 4.5.



**Fig. 4.4 Ultimate load vs Rise**



**Fig.4.5 Ultimate load vs Span /rise ratio ( $\lambda$ )**

From Fig 4.5, it is observed that the ultimate load (Pu) increases with the decrease in span to rise ratio ( $\lambda$ ). From the fig. 4.5 the relationship between Pu and  $\lambda$  can be approximated by the equation (1) where  $\lambda$  value lies ( $5 < \lambda < 20$ ).

$$Pu = -2.244 \lambda^2 + 25.52 \lambda - 19.73 \dots\dots (1)$$

**5.0 CONCLUSIONS**

From the experimental investigations the following conclusions are drawn from the test results.

- i) The deflection of shallow funicular concrete shell decreases with increase in rise within the elastic range.
- ii) The ultimate load carrying capacity increases with the increase in rise of shallow funicular concrete shell.
- iii) The Span/Rise ratio decreases the increment in ultimate load carrying capacity.
- iv) It is concluded that an increment of 11% of deflection (w) is observed in SFS II when compared with SFS I.
- v) It is concluded that an increment of 25 % and 35 % of deflection (w) is observed in SFS III when compared with SFS II and SFS I respectively.

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