

Bunker Consumption Optimization in Liner Shipping: A Metaheuristic Approach

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Abstract-Taking into account increasing volumes of the international seaborne trade, liner shipping companies have to ensure efficiency of their operations in order to remain competitive. The bunker consumption cost constitutes a substantial portion of the total vessel operating cost and directly affects revenues of liner shipping companies. "Slow steaming" became a common strategy among ocean carriers to decrease vessel sailing speeds and reduce bunker consumption costs. However, decreasing vessel sailing speeds may require deployment of more vessels on a given shipping route to provide the agreed service frequency at each port of call. Several bunker consumption optimization methods were developed in the past to capture those conflicting decisions. This paper describes existing bunker consumption optimization methods, outlines their drawbacks, and proposes a new metaheuristic approach. Numerical experiments demonstrate efficiency of the suggested metaheuristic in terms of solution quality and computational time.

Keywords-Maritime Transportation, Liner Shipping, Bunker Consumption Optimization, Metaheuristics

I. INTRODUCTION

The volume of cargo, carried by vessels, significantly increased over the last decades. According to International Chamber of Shipping [1], seaborne trade volumes have quadrupled over the last four decades and increased from 8,000 billion ton-miles in 1968 to 32,000 billion ton-miles in 2008. The heavy fuel oil (HFO) currently costs around \$600 per ton, but is forecasted to increase up to \$1,000 by 2020 [2]. Furthermore, International Maritime Organization (IMO) regulations set a new limit for sulphur oxides (SO_x) content in the fuel outside the emission control areas from 4.50% m/m before January 1, 2012 to 3.50% after January 1, 2012, falling to 0.50% after January 1, 2020 [3]. Such drastic changes in the fuel chemical content requirements will force liner shipping companies to switch from HFO to more expensive marine gas oil (MGO). Currently MGO costs around \$960 per ton, but is projected to increase to \$1,800 by 2020 and to \$2,300 by 2035 [2].

In order to improve transport efficiency liner shipping companies are slowing down their vessels, leading to significant bunker consumption cost savings that may comprise up to 75% of total vessel operational costs [4]. Psaraftis and Kontovas [5] indicate that "slow steaming", when a vessel sails at lower than the designed speed, became a common strategy among liner shipping companies to reduce bunker consumption costs. According to COSCO's first half year earnings statement, "slow steaming" reduced bunker consumption expenses by 18% in the first half of the year 2014 [6]. However, "off-schedule ships, particularly the mega-ships that are slow sailing to save costs, are also a factor...causing port congestion" [7]. Drewry Maritime Research underlined that Asia-Europe route was the least reliable during August-October 2014 with only 58% of vessels arriving within the allocated time window (TW), which is considered as unacceptable for many shippers [8].

Several bunker consumption optimization (BCO) methods were proposed in the literature to assist liner shipping companies in designing efficient vessel schedules. This paper provides an overview of the existing approaches and suggests a new metaheuristic approach. The rest of the paper is organized as follows. The next section provides the problem description, while the third section presents the mathematical formulation. The fourth section overviews the existing BCO methods to solve the considered problem and outlines drawbacks of those methods, while the fifth section describes the developed metaheuristic. The sixth section discusses numerical experiments that were performed in this study, while the last section provides conclusions and future research avenues.

II. PROBLEM DESCRIPTION

The problem of vessel routing and scheduling in liner shipping received a lot of attention from researchers and practitioners, especially during the last ten years. For an excellent review of strategic, tactical and operational decision problems in liner shipping this study refers to Meng et al. [9]. This paper focuses on the vessel schedule design problem, described next.

A. Liner Shipping Route

This study considers a liner shipping route with $I = \{1, \dots, n\}$ ports of call (see Figure 1). Each port is assumed to be visited once¹, and the sequence of visited ports (i.e., port rotation) is already known. The latter decision is made by a liner shipping company at the strategic level [9]. A vessel sails between two subsequent ports i and $i+1$ along leg i . The liner shipping company provides a weekly service at each port of call. The terminal operator at each port sets a specific arrival

¹ This assumption does not limit generality of the suggested methodology and can be relaxed as needed, i.e., some ports can be visited more than once

TW [tw_i^e – the earliest start at port i , tw_i^l – the latest start at port i], during which a vessel should arrive at the port (can be up to 1-3 days depending on the port). Weekly demand (TEUs) at each port is known, while the quantity of containers transported by alliance partners is excluded from the total weekly demand, as this decision is usually made by the liner shipping company at the strategic level [9].

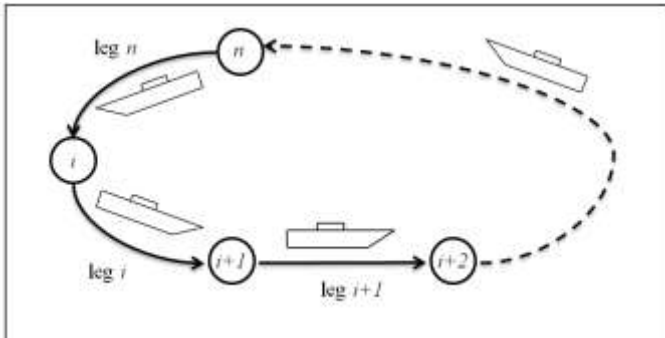


Figure 1. Illustration of a Shipping Route.

B. Vessel Service at Ports of Call

Terminal operators have various contractual agreements with the liner shipping company, according to which each terminal operator offers a set of handling rates $S_i = \{1, \dots, s_i\} \forall i \in I$ to the liner shipping company. If faster service is requested, the port handling time for a given vessel decreases, but the port handling charges, imposed to the liner shipping company, increase. Note that reduced handling time at a port may result in bunker consumption cost savings, since a vessel can sail at a lower speed to the next port of call.

C. Vessel Arrivals

The following scenarios of vessel arrivals will be modeled in this study:

1) If a vessel arrives within a set arrival TW, no penalties will be imposed to the liner shipping company.

2) In certain cases a vessel departing from port i may arrive at the next port $i+1$ before the earliest start tw_{i+1}^e , even when sailing at the lowest possible speed v^{\min} . In such cases it is assumed that the vessel will wait at a dedicated area at port i to ensure arrival within the allocated TW at port $i+1$ ². The port waiting time wt_i can be estimated as $wt_i = tw_{i+1}^e - \frac{l_i}{v_i} - t_i^d$,

where v_i is the sailing speed on leg i , l_i is length of leg i , t_i^d is departure time from port i .

3) If a vessel arrives after the end of the latest start tw_{i+1}^l , monetary penalties are imposed to the liner shipping company (in USD/hr.), but the vessel will still start service upon arrival³. The penalty value is assumed to linearly increase with late arrival hours lt_{i+1} .

² Technically the vessel can also wait at port $i+1$, or split waiting times between ports i and $i+1$

³ It is assumed that the liner shipping company under consideration can negotiate such an agreement

D. Bunker Consumption

It is assumed that a vessel fleet for a given route is homogenous, which is a common practice, as revealed in the literature [10-13]. The relationship between the bunker consumption and the vessel speed is as follows:

$$q(\bar{v}) = q(v^*) \times \left(\frac{\bar{v}}{v^*}\right)^\alpha = \gamma \times (\bar{v})^\alpha \quad (1)$$

where:

$q(\bar{v})$ – daily bunker consumption (tons of fuel/day);

\bar{v} – average daily sailing speed (knots);

$q(v^*)$ – daily bunker consumption when sailing at the designed speed (tons of fuel/day);

v^* – average daily sailing speed (knots);

α, γ – coefficients calibrated from the historical data;

Generally, an additional regression analysis should be conducted to determine the values of α and γ for each vessel in the fleet [11, 14, 15]. Due to lack of data, the most common values from the literature [5, 11] are adopted in this paper (i.e., $\alpha=3$ and $\gamma=0.012$). Once the liner shipping company decides on a sailing speed between consecutive ports, it is assumed to remain constant. Factors affecting the vessel speed during voyage (e.g., weather conditions, wind speed, height of waves, etc.) are not considered. The fuel consumption by auxiliary engines was included in the weekly vessel operating cost.

Note that bunker consumption per nautical mile $f(v_i)$ at leg i can be estimated as follows:

$$\begin{aligned} f(v_i) &= q(v_i) \times \left(\frac{t_i}{24}\right) \times \frac{1}{l_i} = \gamma \times (v_i)^\alpha \times \left(\frac{l_i}{24 \times v_i}\right) \times \frac{1}{l_i} \\ &= \frac{\gamma \times (v_i)^{\alpha-1}}{24} \quad \forall i \in I \end{aligned} \quad (2)$$

where:

t_i – sailing time between ports i and $i+1$ (hrs.)

E. Decisions

The problem, considered in this study, can be classified as a tactical level problem and will be referred to as the vessel schedule design problem. In this problem the liner shipping company determines the following:

1) Number of vessels assigned to the given route in order to provide weekly service at each port (decision on fleet size and mix is assumed to be made at the strategic level, 9);

2) Handling time (or handling rates) at each port, taking into account TW constraints and increasing charges for faster service at each port;

3) Port waiting time to ensure feasibility of arrival at the next port of call;

4) Sailing speed between consecutive ports, taking into account TW constraints at each port and associated bunker consumption costs;

5) Vessel late arrival fees.

A liner shipping company sets a maximum quantity of vessels that can be deployed at any given route ($q \leq q^{\max}$) and sets limits on lower and upper vessel sailing speed ($v^{\min} \leq v_i \leq v^{\max} \forall i \in I$). The minimum sailing speed v^{\min} is selected to reduce wear of the vessel's engine [16], while the maximum sailing speed v^{\max} is defined by the capacity of the vessel's engine [5]. Note that all decisions are interrelated. Selecting lower sailing speed reduces bunker consumption, but may require deployment of more vessels at the given route to ensure that weekly service is met, which increases the total weekly operating cost (e.g., crew costs, maintenance, repairs, insurance, etc.). Various port handling rates further allow the liner shipping company to weigh different options between sailing and port handling times (e.g., faster handling rate reduces the service time at a given port, which may allow sailing at a lower speed to the next port of call). On the other hand, higher handling rates may not always be favorable as they may lead to the vessel waiting once service is completed.

III. MODEL FORMULATION

This section presents a mixed integer non-linear mathematical model for the vessel schedule design problem **VSDP** with variable vessel sailing speeds and port handling times.

Nomenclature

Sets

- $I = \{1, \dots, n\}$ set of ports to be visited
- $S_i = \{1, \dots, s_i\}$ set of available handling rates⁴ at port i

Decision variables

- $v_i \forall i \in I$ vessel sailing speed at leg i , connecting ports i and $i+1$
- $x_{is} \forall i \in I, s \in S_i$ =1 if handling rate s is selected at port i (=0 otherwise)

Auxiliary variables

- q number of vessels deployed at the given route
- $t_i^a \forall i \in I$ arrival time at port i (hrs.)
- $t_i^d \forall i \in I$ departure time from port i (hrs.)
- $wt_i \forall i \in I$ hours of waiting time at port i (hrs.)
- $t_i \forall i \in I$ vessel sailing time at leg i , connecting ports i and $i+1$ (hrs.)
- $f(v_i) \forall i \in I$ bunker consumption at leg i at sailing speed v_i (tons of fuel/nmi)
- $lt_i \forall i \in I$ hours of late arrival at port i (hrs.)

Parameters

- β unit bunker cost (USD/ton)
- c^{OC} vessel weekly operating cost (USD/week)
- c^{LT} hourly delayed arrival penalty (USD)

- $l_i \forall i \in I$ length of leg i (nmi)
- v^{\min} minimum vessel sailing speed (knots)
- v^{\max} maximum vessel sailing speed (knots)
- q^{\max} number of available vessels
- $p_{is} \forall i \in I, s \in S_i$ vessel handling time at port i under handling rate s (hrs.)
- $tw_i^e \forall i \in I$ the earliest start at port i (hrs.)
- $tw_i^l \forall i \in I$ the latest start at port i (hrs.)
- $sc_{is} \forall i \in I, s \in S_i$ handling cost at port i under handling rate s (USD/hrs.)

VSDP:

$$\min [c^{OC}q + \beta \sum_{i \in I} l_i f(v_i) + \sum_{i \in I} \sum_{s \in S_i} p_{is} x_{is} sc_{is} + \sum_{i \in I} c^{LT} lt_i] \quad (3)$$

Subject to:

$$\sum_{s \in S_i} x_{is} = 1 \forall i \in I \quad (4)$$

$$t_i = \frac{l_i}{v_i} \forall i \in I \quad (5)$$

$$t_i^a \geq tw_i^e \forall i \in I \quad (6)$$

$$t_i^a + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i + t_i \geq tw_{i+1}^e \forall i < |I| \quad (7)$$

$$t_i^a + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i + t_i - 168q \geq tw_{i+1}^e \forall i = |I| \quad (8)$$

$$t_i^d = t_i^a + \sum_{s \in S_i} (p_{is} x_{is}) + wt_i \forall i \in I \quad (9)$$

$$lt_i \geq t_i^d - tw_i^l \forall i \in I \quad (10)$$

$$t_{i+1}^a = t_i^d + t_i \forall i < |I| \quad (11)$$

$$t_1^a = t_i^d + t_i - 168q \forall i = |I| \quad (12)$$

$$168q \geq \sum_{i \in I} t_i + \sum_{s \in S_i} (p_{is} x_{is}) + \sum_{i \in I} wt_i \quad (13)$$

$$q \leq q^{\max} \quad (14)$$

$$v^{\min} \leq v_i \leq v^{\max} \forall i \in I \quad (15)$$

$$x_{is} \in \{0,1\} \quad (16)$$

$$q, q^{\max} \in N \quad (17)$$

$$v_i, t_i^a, t_i^d, wt_i, t_i, f(v_i), lt_i, \beta, c^{OC}, c^{LT}, l_i, v^{\min}, v^{\max}, p_{is}, tw_i^e, tw_i^l, sc_{is} \in R^+ \forall i \in I, s \in S_i \quad (18)$$

The objective function (3) minimizes the total route service cost, which includes 4 components: 1) total vessel weekly operating cost, 2) total bunker consumption cost, 3) total port handling cost, and 4) total late arrival penalty. Constraints set (4) indicate that only one handling rate can be selected at each port of call. Constraints set (5) calculate a vessel sailing time between ports i and $i+1$. Constraints set (6) ensure that a vessel cannot arrive at port $i \in I$ before the earliest start. Constraints sets (7) and (8) compute waiting time at port $i \in I$, necessary to ensure feasibility of arriving at the next port of call. Constraints set (9) calculate a vessel departure time from

⁴ Set of handling rates contains indexes of available handling rates (i.e., if a terminal operator at port i offers two handling rates 75 TEUs/hr. and 50 TEUs/hr., then $S_i = \{1, 2\}$)

port $i \in I$. Constraints set (10) estimate hours of late arrival at port $i \in I$. Constraints sets (11) and (12) compute a vessel arrival at the next port of call $(i+1) \in I$. Constraints set (13) ensure weekly service frequency (168 denotes the total number of hours in a week). The right-hand-side of an equality estimates the total turnaround time of a vessel at the given route (where the first component is the total sailing time, the second component is the total port handling time, and the third component is the total port waiting time). Constraints set (14) ensure that the number of vessels to be deployed at the given route should not exceed the number of available vessels. Constraints set (15) show that a vessel sailing speed should be within specific limits. Constraints (16) – (18) define ranges of parameters and variables.

IV. STATE OF THE ART

One of the difficulties in solving tactical level problems in liner shipping is non-linearity of the bunker consumption

function (the second component of the *VSDP* objective). There exist several methods to address this issue:

1) Enumeration Method (M1) – it is assumed that sailing speed remains constant throughout the voyage, i.e. does not change from leg to leg [17, 18]. In this case the required sailing speed can be determined based on the number of vessels deployed at the given route, the total port handling and waiting times, and required frequency of service (using eq. (13) in *VSDP*). The total cost (estimated as a sum of bunker consumption and operating costs) should be computed for all possible scenarios by varying the number of deployed vessels (typically does not exceed 20-25), and then the best alternative should be selected (see Figure 2A).

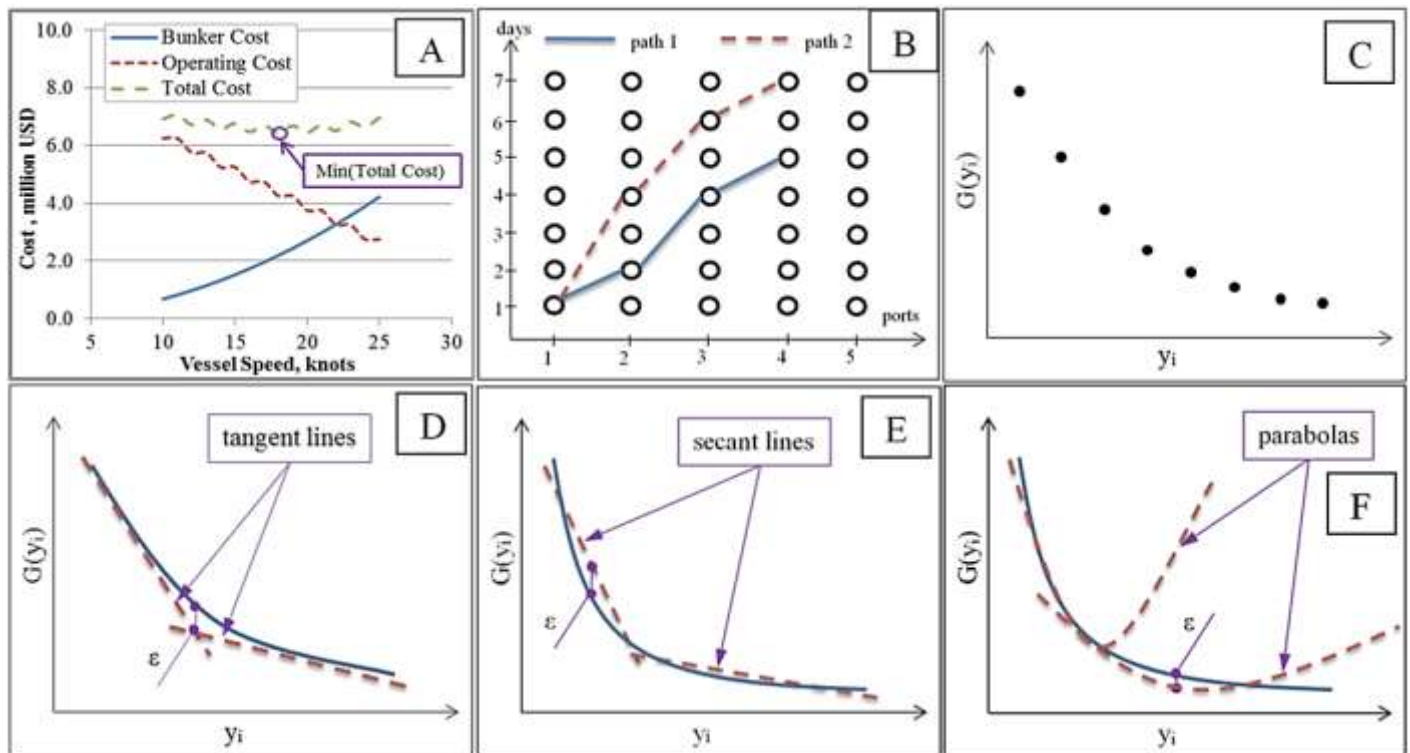


Figure 2. Existing BCO Methods.

2) Dynamic Programming Method (M2) – requires construction of a time-space network, where the x-axis represents ports to be visited (see Figure 2B) and y-axis represents time, discretized in days. Once the time-space network is created, the problem of selecting vessel speed and quantity of deployed vessels can be reduced to the shortest path problem [19, 20].

3) Discretization Method (M3) – vessel sailing speed reciprocal $y_i = \frac{1}{v_i}$ is discretized (see Figure 2C), and the bunker consumption $G(y_i)$ is estimated for each value of y_i [15, 21].

4) Tailored Methods (M4) – a linear or quadratic approximation is used for the non-linear bunker consumption

function [11, 12, 16]. The linear approximation can be represented either by a set of tangent (see Figure 2D) or secant lines (see Figure 2E). The quadratic approximation can be represented by a set of parabolas (see Figure 2F). Approximations can be static or dynamic. Static approximations have a predetermined number of elements (either lines or parabolas). Dynamic approximations require generating additional elements if after solving linear or quadratic *VSDP* the approximation error ϵ exceeds a target limit.

5) Second Order Conic Programming (SOCP) Method (M5) – requires transformation of the original *VSDP* formulation to the SCOP formulation [14].

For a detailed review of BCO methods this study refers to Wang et al. [16]. Drawbacks of the existing methods can be summarized as follows:

1) In reality it is highly unlikely that a vessel will sail at the same speed throughout the whole voyage as assumed by M1. The vessel may be required to wait at the given port of call for a significant amount of time in order to arrive at the next port of call within the allocated TW.

2) Higher time discretization rate (e.g., discretize every 4 hrs. instead of discretizing every day, see Figure 2B) will increase the number of nodes in the time-space network, which in turn will increase the computational time of M2. Furthermore, the optimal solution, which lies between two nodes (e.g., 8.7 hrs. lying between 8 hrs. and 12 hrs. in case of 4 hrs. discretization rate), will never be discovered.

3) Similar to M2, higher speed discretization rate will increase the computational time of M3.

4) All tailored methods have an approximation error ε . The error can be reduced by increasing number of elements in the approximation. Dynamic approximations require solving linearized or quadratic *VSDP* more than once.

5) M5 requires a mixed integer SOCP solver for *VSDP* with SOCP constraints.

V. SOLUTION APPROACH

A Memetic Algorithm (*MA*) was developed to address drawbacks of the existing BCO methods, described in the previous section. *MAs* belong to the group of Evolutionary Algorithms (*EAs*), and are widely used for solving complex problems in different fields [22]. However, while *EAs* construct individuals using stochastic operators, *MAs* employ along with the stochastic operators local search heuristics and generally provide higher quality solutions and faster convergence [22]. Advantages of the proposed *MA* to existing BCO methods are as follows:

1) Does not restrict the vessel sailing speed to remain constant throughout the voyage (unlike M1);

2) Considers the entire search space (unlike M2 and M3, which discretize the search space);

3) Estimates bunker consumption using the non-linear function without generating approximations, which have a certain error ε (unlike M4);

4) Does not require any specific solvers and transformation of *VSDP* formulation (unlike M5);

The main steps of the developed *MA* are outlined in Procedure 1. In steps 1- 4 the chromosomes and population are initialized. In step 5 fitness of the initial population is evaluated. Then, the algorithm enters the main loop (steps 7 through 11). In step 8, function *SelectParents* (Pop_{gen}) identifies parents from the current population (i.e., variable $Parents_{gen}$) to be used in step 9 and produce new offspring. In step 9, function *MAoperation* ($Parents_{gen}, MutRate$) applies a custom operator, which uses a stochastic search and a local search heuristic (*LSH*) to produce new offspring (i.e., variable $Offspring_{gen}$). In step 10, function *Evaluate* ($Offspring_{gen}$) calculates fitness function values (i.e., variable Fit_{gen}) for the offspring, and in step 11, function *Select* ($Offspring_{gen}, Fit_{gen}$) selects individuals, based on their fitness, to become candidate parents in the next generation. *MA* exits the loop, when a termination criterion is satisfied. The algorithm was coded in MATLAB R2014a. Next this section presents in detail each of the components of the developed *MA*.

Procedure 1. Memetic Algorithm

MA($I, S_i, PopSize, MutRate, StopCriterion$)

in: $I = \{1, \dots, n\}$ - set of ports to be visited; $S_i = \{1, \dots, s_i\}$ - set of available rates at each port; $PopSize$ - population size; $MutRate$ - mutation rate; $StopCriterion$ - stopping criterion

out: *BestChrom* - the best vessel schedule

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1:  $|Pop| \leftarrow PopSize; |Fit| \leftarrow PopSize; |Parents| \leftarrow PopSize; |Offspring| \leftarrow PopSize;$                                  $\triangleleft$  Initialization
2:  $Chrom \leftarrow InitChrom(I, S_i)$                                                                      $\triangleleft$  Chromosome initialization
3:  $gen \leftarrow 0$ 
4:  $Pop_{gen} \leftarrow InitPop(Chrom, PopSize)$                                                              $\triangleleft$  Population initialization
5:  $Fit_{gen} \leftarrow Evaluate(Pop_{gen})$                                                                      $\triangleleft$  Initial population fitness evaluation
6: while  $StoppingCriterion \leftarrow FALSE$  do
7:    $gen \leftarrow gen + 1$ 
8:    $Parents_{gen} \leftarrow SelectParents(Pop_{gen})$                                                          $\triangleleft$  Select parents
9:    $Offspring_{gen} \leftarrow MAoperation(Parents_{gen}, MutRate)$                                            $\triangleleft$  Produce offspring
10:   $Fit_{gen} \leftarrow Evaluate(Offspring_{gen})$                                                              $\triangleleft$  Offspring fitness evaluation
11:   $Pop_{gen+1} \leftarrow Select(Offspring_{gen}, Fit_{gen})$                                                  $\triangleleft$  Define population in the next generation
12: end while
13: return BestChrom

```

A. Chromosome Representation

A real valued chromosome was used in the developed **MA** to represent a solution (i.e., vessel schedule). An example of a chromosome is presented in Figure 3 for a liner shipping route with 8 ports of call. For each port the chromosome includes the following information: a) arrival time, b) sailing speed to the next port, c) handling rate selected, and d) waiting time. For example, according to the schedule the vessel should arrive at the 3rd port of call on Thursday at 2:42 PM (i.e., $3 \times 24 + 14.7 = 86.7$ hrs.), be served at the port under handling rate of 100 TEUs/hr., wait for 14.7 hrs. after service completion, and sail to the next port of call at the speed of 24.0 knots.

1	2	3	4	5	6	7	8	← ports
0.0	63.4	86.7	93.6	131.1	152.8	184.2	234.1	← arrival time (hrs.)
15.1	12.6	24.0	18.6	13.4	21.8	24.6	12.0	← sailing speed (knots)
125	30	100	100	50	75	125	30	← handling rate (TEUs/hr)
15.1	21.2	14.7	24.1	47.6	18.9	40.9	6.0	← waiting time (hrs.)

Figure 3. Chromosome Representation.

B. Chromosome and Population Initialization

Two **LSHs** are used to initialize the population. The first **LSH** is directed to minimize the total number of deployed vessels at the given route by selecting the maximum handling rate (i.e., minimum port time) at each port of call and the maximum possible sailing speed between ports, taking into account TW constraints. The first **LSH** will be referred to as Minimizing the Number of Deployed Vessels Heuristic (**MINDVH**). The second **LSH** is directed to maximize the total number of deployed vessels at the given route by selecting the minimum handling rate (i.e., maximum port time) at each port of call and the minimum possible sailing speed between ports, taking into account TW constraints. The second **LSH** will be referred to as Maximizing the Number of Deployed Vessels Heuristic (**MANDVH**). **LSHs** are outlined in Procedures 2 and 3.

Procedure 2. Minimizing the Number of Deployed Vessels Heuristic

MINDVH ($I, S_i, l_i, tw_i^l, tw_i^e, v^{\min}, p_{is}$)

in: $I = \{1, \dots, n\}$ - set of ports to be visited; $S_i = \{1, \dots, s_i\}$ - set of available rates at each port; l_i - length of leg i ; tw_i^e - the earliest start at port i ; tw_i^l - the latest start at port i ; v^{\min} - minimum sailing speed; p_{is} - vessel handling time at port i under handling rate s (hrs.)

out: t^a - arrival time at each port of call; v - vessel sailing speed; p - handling time at each port of call; wt - waiting time at each port of call;

- 1: $|t^a| \leftarrow n; |v| \leftarrow n; |p| \leftarrow n; |wt| \leftarrow n$ ◁ Initialization
- 2: $i \leftarrow 1; t_1^a \leftarrow tw_1^e$
- 3: **for all** $i \in I$ **do**
- 4: $p_i \leftarrow \min(p_{is})$ ◁ Select handling time
- 5: **if** $\frac{l_i}{tw_{i+1}^l - (t_i^a + p_i)} \leq v^{\min}$ **then**
- 6: $wt_i \leftarrow tw_{i+1}^l - \frac{l_i}{v^{\min}} - (t_i^a + p_i)$ ◁ Estimate waiting time
- 7: **else**
- 8: $wt_i \leftarrow 0$
- 9: **end if**
- 10: $v_i \leftarrow \frac{l_i}{tw_{i+1}^l - (t_i^a + p_i + wt_i)}$ ◁ Estimate vessel sailing speed
- 11: $t_{i+1}^a \leftarrow t_i^a + p_i + wt_i + \frac{l_i}{v_i}$ ◁ Estimate arrival time at the next port of call
- 14: $i \leftarrow i + 1$
- 15: **end for**
- 16: **return** t^a, v, p, wt

Half of the population will be initialized using **MINDVH**, while the other half will be initialized using **MANDVH**. Preliminary **MA** runs were performed to select the population size *PopSize*, and results will be presented in the numerical experiments section.

C. Parent Selection

Parent selection determines individuals from the current population that will be allowed to produce offspring via **MA** operations at a given generation. The proposed **MA** applies a

deterministic parent selection scheme (i.e., all survived offspring become parents), as this strategy is widely used in Evolutionary Programming and Genetic Algorithms [22].

Procedure 3. Maximizing the Number of Deployed Vessels Heuristic

MANDVH ($I, S_i, l_i, tw_i^e, tw_i^l, v^{\min}, p_{is}$)

in: $I = \{1, \dots, n\}$ - set of ports to be visited; $S_i = \{1, \dots, s_i\}$ - set of available rates at each port; l_i - length of leg i ; tw_i^e - the earliest start at port i ; tw_i^l - the latest start at port i ; v^{\min} - minimum sailing speed; p_{is} - vessel handling time at port i under handling rate s (hrs.)

out: t^a - arrival time at each port of call; v - vessel sailing speed; p - handling time at each port of call; wt - waiting time at each port of call;

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1:  $|t^a| \leftarrow n; |v| \leftarrow n; |p| \leftarrow n; |wt| \leftarrow n$                                 < Initialization
2:  $i \leftarrow 1; t_1^a \leftarrow tw_1^e$ 
3: for all  $i \in I$  do
4:    $p_i \leftarrow \max(p_{is})$                                                     < Select handling time
5:   if  $\frac{l_i}{tw_{i+1}^l - (t_i^a + p_i)} \leq v^{\min}$  then
6:      $wt_i \leftarrow tw_{i+1}^l - \frac{l_i}{v^{\min}} - (t_i^a + p_i)$                             < Estimate waiting time
7:   else
8:      $wt_i \leftarrow 0$ 
9:   end if
10:   $v_i \leftarrow \frac{l_i}{tw_{i+1}^l - (t_i^a + p_i + wt_i)}$                                 < Estimate vessel sailing speed
11:   $t_{i+1}^a \leftarrow t_i^a + p_i + wt_i + \frac{l_i}{v_i}$                                 < Estimate arrival time at the next port of call
14:   $i \leftarrow i + 1$ 
15: end for
16: return  $t^a, v, p, wt$ 
    
```

D. MA Operations

A custom **MA** operator was developed in this study to produce offspring. The **MA** operation will be performed for randomly selected ports of call, belonging to the given shipping route (stochastic search). The mutation rate *MutRate* defines the number of ports, which will undergo the **MA** operation. For selected ports the **MA** operator will apply **LSH**. An example of the **MA** operation at port i is presented in Figure 3, where the **MA** operator performs the following steps:

- 1) Generate a number of candidate solutions $d \in D$ with i) arrival times, satisfying TW constraint at port i : $t_{id}^a = tw_i^e + (tw_i^l - tw_i^e)rand$, where *rand* is a random number between 0 and 1, and ii) randomly selected port handling rate from the available handling rates at port i ;
- 2) Flip the coin and get the value between 0 and 1;
- 3) If the coin value is zero, select new arrival time t_{id}^a and handling rate r_i , which minimize the bunker consumption cost from sailing between ports $i-1$ and $i+1$ and the handling cost at port i (i.e., find the shortest path between ports $i-1$ and $i+1$);

Else, select new arrival time t_{id}^a and handling rate r_i , which maximize the bunker consumption cost from sailing between ports $i-1$ and $i+1$ and the handling cost at port i (i.e., find the longest path between ports $i-1$ and $i+1$);

- 4) Update the chromosome based on new arrival time and handling rate.

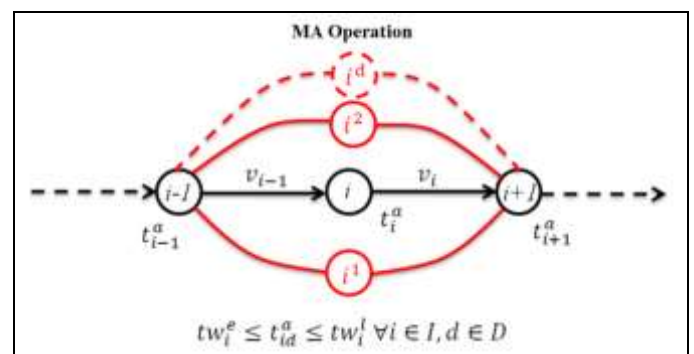


Figure 4. **MA** Operation.

Note that by selecting the shortest path between ports $i-1$ and $i+1$ the **MA** operator might increase the number of vessels

to be deployed at the given route and vice versa. The number of candidate solutions, generated at each mutated port of call, will be referred to as discretization rate *DisRate*. Preliminary **MA** runs were performed to select *MutRate* and *DisRate* values, and results will be presented in the numerical experiments section.

E. Fitness Function

For **EAs/MAs** the fitness function is usually associated with the objective function [22]. In the proposed **MA** the fitness function value was set equal to the objective function value without applying any scaling mechanisms.

F. Offspring Selection

The offspring selection at a given generation of a **MA** is an important part of its design [22]. The Tournament Selection is used in the developed **MA**. The Tournament Selection has two parameters: a) tournament size (*TourSize*) – number of individuals participating in each tournament, and b) individuals selected (*IndSel*) – number of individuals selected based on their fitness in each tournament to become candidate parents in the next generation. Preliminary **MA** runs were performed to select values of those parameters and results will be presented in the numerical experiments section. The Elitist Strategy [22] is applied in the developed **MA** to ensure that the best individual lives more than one generation.

G. Stopping Criterion

In this paper the algorithm was terminated, if no change in the objective function value was observed after a pre-specified number of generations (*MaxNumGen* of 300 generations) or the maximum number of generations was reached (*LimitGen* of 500 generations).

H. Comparison to the Existing Method

Performance of the developed **MA** was assessed based on comparison to a static secant approximation method (see Figure 2E). **VSDP** formulation was linearized by replacing sailing speed v_i with its reciprocal y_i and the non-linear bunker consumption function $G(y)$ with its secant approximation $\overline{G}_m(y)$, where m – is the number of segments. Let $K = \{1, \dots, m\}$ be the set of linear segments of the piecewise function $\overline{G}_m(y)$. Let $b_{ik} = 1$ if segment k is selected for approximation of the bunker consumption function at leg i (=0 otherwise). Denote as $st_k, ed_k, k \in K$ the speed reciprocal values at the start and end (respectively) of linear segment k ; $IN_k, SL_k, k \in K$ the slope and an intercept of linear segment k (obtained from a piecewise linear regression analysis); and M_1, M_2 as sufficiently large positive numbers. Then linearized **VSDP** (**VSDPL**) can be reformulated as a linear problem as follows.

VSDPL:

$$\min [c^{OC} q + \beta \sum_{i \in I} l_i \sum_{k \in K} \overline{G}_k(y_i) + \sum_{i \in I} \sum_{s \in S_i} p_{is} x_{is} sc_{is} + \sum_{i \in I} c^{LT} t_i] \quad (19)$$

Subject to:

Constraint sets (4), (6)-(14), (16)-(18)

$$\sum_{k \in K} b_{ik} = 1 \forall i \in I \quad (20)$$

$$st_k b_{ik} \leq y_i \forall i \in I, k \in K \quad (21)$$

$$ed_k + M_1(1 - b_{ik}) \geq y_i \forall i \in I, k \in K \quad (22)$$

$$\overline{G}_k(y_i) \geq SL_k y_i + IN_k - M_2(1 - b_{ik}) \forall i \in I, k \in K \quad (23)$$

$$t_i = l_i y_i \forall i \in I \quad (24)$$

$$\frac{1}{v_{\max}} \leq y_i \leq \frac{1}{v_{\min}} \forall i \in I \quad (25)$$

In **VSDPL** constraints set (20) ensure that only one segment k will be selected for approximation of the bunker consumption function at leg i . Constraints sets (21) and (22) define range of vessel sailing speed reciprocal values, when segment k is selected for approximation of the bunker consumption function at leg i . Constraints set (23) estimate the approximated bunker consumption at leg i . Constraints set (24) calculate a vessel sailing time between ports i and $i+1$. Constraints set (25) show that a reciprocal of vessel sailing speed should be within specific limits. Positive number M_1 was introduced to ensure that each segment $k \in K$ of $\overline{G}_k(y)$ function approximates a non-linear function $G_k(y)$ only for a specific range of y . Positive number M_2 was introduced to estimate the approximated bunker consumption value $\overline{G}_k(y)$ for a given y . Strict lower bounds for M_1 and M_2 can be defined as follows: $M_1 = \frac{1}{v_{\min}}$, $M_2 = SL_k(\frac{1}{v_{\max}}) + IN_k$. **VSDPL** can be solved efficiently using CPLEX. Comparison of **VSDPL** and **MA** will be discussed in the numerical experiments section.

VI. NUMERICAL EXPERIMENTS

This section presents a number of numerical experiments conducted to evaluate performance of the developed **MA**.

A. Input Data Description

This study will consider the French Asia Line 1 route, served by CMA CGM liner shipping company (see Figure 5⁵). This route connects North Europe, North Africa, Malta, Middle East Gulf, and Asia. The port rotation for French Asia Line 1 route includes 15 ports of call (distance to the next port of call in nautical miles is presented in parenthesis, estimated using world seaports catalogue⁶):

1. Southampton, GB (571) → 2. Hamburg, DE (341) → 3. Rotterdam, NL (119) → 4. Zeebrugge, BE (302) → 5. Le Havre, FR (2538) → 6. Malta, MT (4089) → 7. Khor Al Fakkan, AE (6397) → 8. Ningbo, CN (87) → 9. Shanghai, CN (606) → 10. Xiamen, CN (432) → 11. Hong Kong, HK (407) → 12. Chiwan, CN (395) → 13. Yantian, CN (2045) → 14. Port Kelang, MY (7490) → 15. Tanger Med, MA (1367) → 1. Southampton, GB

⁵ <http://www.cma-cgm.com/products-services/line-services/flyer/FAL> (accessed on 03/07/2015)

⁶ <http://ports.com/sea-route>



Figure 5. French Asia Line 1.

The required numerical data were generated based on the available liner shipping literature and are presented in Table 1. The latest start at each port of call was set using the following relationship: $tw_i^l = tw_{i-1}^l + \frac{l_i}{U[v^{\min}; v^{\max}]}$, where U denotes uniformly distributed pseudorandom numbers. The duration of a TW ($tw_i^l - tw_i^e$) was assigned as $U[24;72]$ hrs. [23]. A set of available port handling times p_{is} at each port of call was assigned based on the weekly demand NC_i (in TEUs) and the available handling rates S_i at the given port. Large ports were assumed to have the weekly demand, uniformly distributed between 500 TEUs and 2000 TEUs. Note that term “large port” was applied to those ports of call, if they were included in the list of top 20 world container ports based on their throughput [24]. Weekly demand for smaller ports was uniformly distributed between 200 TEUs and 1000 TEUs. Large ports were able to offer 4 possible handling rates: [125; 100; 75; 50] TEUs/hr. Smaller ports could provide either 3 ([100; 75; 50] TEUs/hr.) or 2 handling rates ([75; 50] TEUs/hr.). The latter assumption can be explained by the fact that terminal operators at large ports usually have more vessel handling equipment available and can offer more handling rate options to the liner shipping company. Furthermore, higher amounts of TEUs handled can increase productivity.

TABLE I. NUMERICAL DATA

Bunker consumption coefficients α, γ	$\alpha=3, \gamma=0.012$
Unit bunker cost β (USD/ton)	500
Vessel weekly operating cost c^{OC} (USD/week)	300,000
Delayed arrival penalty c^{LT} (USD/hr.)	$U[5,000; 10,000]$
Minimum vessel sailing speed v^{\min} (knots)	10
Maximum vessel sailing speed v^{\max} (knots)	25
Maximum number of deployed vessels q^{\max}	20
TW duration (hrs.)	$U[24; 72]$

The handling cost at each port i under handling rate s was computed as: $sc_{is} = asc \pm U[0;50] \forall i \in I, s \in S_i$, where asc is the average container handling cost. Based on the available literature [25-28] and assuming a mix of vessel operations that include mooring, loading and discharge of containers, type of

container (empty, loaded, size, reefer), re-stowing (on-board the vessel or via quay), the average container handling cost was set equal to [700; 625; 550; 475] USD/TEU for handling rates [125; 100; 75; 50] TEUs/hr. respectively. It was assumed that each terminal operator perceives handling cost differently (i.e., service charge for the same handling rate varies from port to port), which is accounted for by the second (and random) term of the sc_{is} formula.

All numerical experiments were conducted on a Dell T1500 Intel(T) Core i5 Processor with 1.96 GB of RAM. A static secant approximation for the bunker consumption function was developed using MATLAB 2014a. A linearized mixed-integer problem formulation **VSDPL** was solved using CPLEX of General Algebraic Modeling System (GAMS).

B. MA Parameter Tuning

Based on preliminary **MA** runs the following values will be set for **MA** parameters: 1) *PopSize* = 40, 2) *MutRate* = 2, 3) *DisRate* = 10, 4) *TourSize* = 15, and 5) *IndSel* = 10.

C. MA Performance

A total of 20 instances were developed using the data, described in the beginning of section VI and presented in Table 1, by changing arrival TW at each port of call. The developed **MA** was compared to **VSDPL**, which applies the static secant approximation with $m=100$ segments for the bunker consumption function linearization. Results for all 20 problem instances are presented in Table 2, where columns 1 through 8 show the following: 1) problem instance; 2) true value of the objective function Z at the optimal solution (i.e., value of the non-linear objective function at the solution, provided by **VSDPL**); 3) objective function value, provided by **VSDPL**; 4) average over 10 replications objective function value, provided by **MA**; 5) **VSDPL** gap $\Delta = \frac{Z - Z(\text{VSDPL})}{Z}$;

6) **MA** gap $\Delta = \frac{Z - Z(\text{MA})}{Z}$; 7) average over 10 replications **VSDPL** computational time $t(\text{VSDPL})$; 8) average over 10 replications **MA** computational time $t(\text{MA})$.

TABLE II. MA PERFORMANCE

Instance	Z, 10 ⁶ USD	Z(VSDPL), 10 ⁶ USD	Z(MA), 10 ⁶ USD	Δ (VSDPL)	Δ (MA)	t(VSDPL), sec	t(MA), sec
I1	12.76	12.46	13.29	2.4%	-4.1%	30.4	19.9
I2	15.15	14.85	15.28	2.0%	-0.9%	28.0	18.4
I3	14.39	14.09	14.90	2.1%	-3.6%	27.6	18.0
I4	15.10	14.80	15.43	2.0%	-2.2%	27.1	18.5
I5	14.68	14.38	15.23	2.0%	-3.7%	29.1	18.2
I6	15.26	14.96	15.50	2.0%	-1.6%	26.8	18.4
I7	14.92	14.62	15.33	2.0%	-2.7%	28.6	17.9
I8	14.97	14.67	15.07	2.0%	-0.7%	28.5	18.3
I9	15.05	14.75	15.40	2.0%	-2.3%	28.4	18.3
I10	14.82	14.52	15.17	2.0%	-2.3%	28.3	18.4
I11	15.01	14.71	15.19	2.0%	-1.2%	26.8	18.5
I12	14.63	14.33	15.18	2.1%	-3.8%	26.5	18.3
I13	15.20	14.90	15.42	2.0%	-1.4%	28.3	18.4
I14	15.15	14.85	15.34	2.0%	-1.3%	28.9	18.2
I15	14.73	14.43	14.80	2.0%	-0.4%	34.2	18.0
I16	14.92	14.62	15.62	2.0%	-4.7%	27.8	18.4
I17	14.91	14.61	15.13	2.0%	-1.5%	29.8	18.5
I18	14.57	14.27	14.77	2.1%	-1.3%	26.1	18.6
I19	15.34	15.04	15.52	2.0%	-1.2%	28.6	18.7
I20	15.12	14.82	15.24	2.0%	-0.8%	26.0	18.2

MA optimality gap is larger than VSDPL optimality gap
 MA optimality gap is smaller than VSDPL optimality gap

Average over 10 replications absolute optimality gap values for VSDPL and MA are presented in Figure 6 for all 20 problem instances.

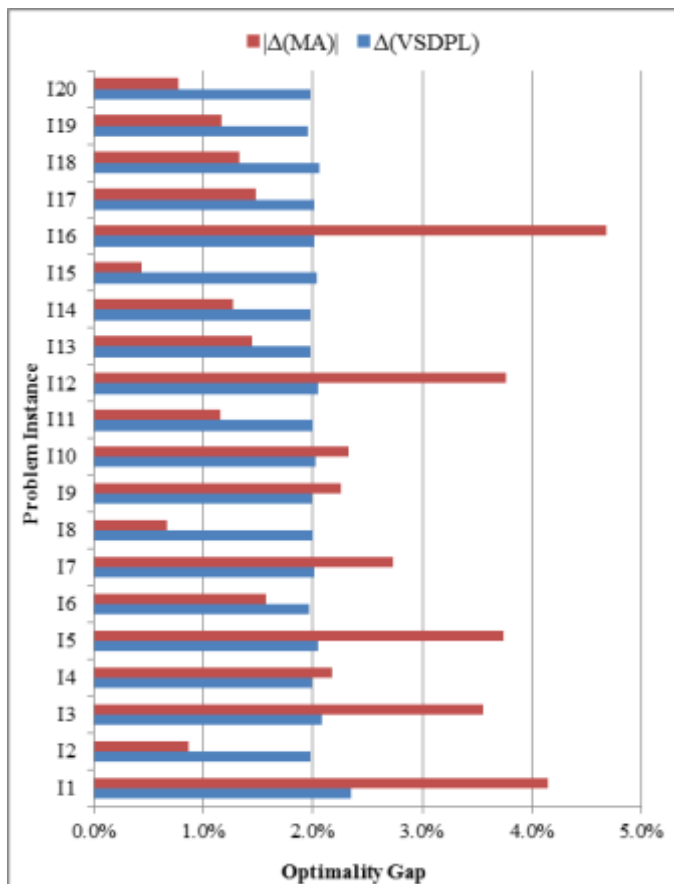


Figure 6. Absolute Optimality Gap Values for VSDPL and MA

Results from numerical experiments indicate that on average VSDPL underestimates Z by 2.0%, while MA on average provides the objective function value higher than Z by 2.1%. Results suggest that for approximately 50% of instances

MA returned the objective function value closer to Z as compared to VSDPL. Furthermore, MA outperformed VSDPL in terms of computational time on average by 54% over all instances and replications. Results of the conducted numerical experiments demonstrate that the developed MA can serve as an efficient BCO tool for liner shipping companies.

VII. CONCLUSIONS AND FUTURE RESEARCH AVENUES

This paper proposed a new metaheuristic approach to assist liner shipping companies with design of efficient vessel schedules. The developed metaheuristic has a number of advantages over the existing bunker consumption optimization methods: 1) does not restrict the vessel sailing speed to remain constant throughout the voyage; 2) considers the entire search space; 3) estimates bunker consumption using the non-linear function without generating approximations; and 4) does not require any specific solvers. Numerical experiments were conducted for French Asia Line 1 route, served by CMA CGM liner shipping company. Performance of the developed Memetic Algorithm was evaluated against one of the existing methods, which applied a static secant approximation to linearize the bunker consumption function. Results demonstrated the efficiency of the proposed algorithm in terms of solution quality and computational time. The scope of future research may focus on the following: a) uncertainty in port handling and sailing times, b) multiple service routes, c) heterogeneous vessel fleet, and d) multiple (non-consecutive) service time windows at each port of call.

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