A Constraint Programming and Hybrid Approach to Nurse Rostering Problems

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Abstract:- This paper describes a decision support methodologies for nurse rostering problem in a modern hospital environment. In particular, it is very important to efficiently utilise time and effort, to evenly balance the workload among people and to attempt to satisfy personnel preference. We presented a complete model to formulate all the complex real-world constraints, solution approach and Hybrid approaches to nurse rostering problem.

Keywords: *Constraints programming, hybrid, LPP, IPP , Nurse rostering*

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1. Introduction- Nurse Rostering Problem

This paper introduces application of Nurse Rostering Problem (NRP) by the integration methods. Firstly we present a brief overview of the modelling issues of NRP. Then the solution approaches to the NRP, categorised the methodologies used. There is no formal definition of NRP due to the fact that a variety of the problems are present in the application. Usually the problem is described informally. In the description of NRP, some of the key terms and expressions are frequently used in literature. Here we described a distinction between schedule and roster. In this process, we present a line-of-work for a nurse within the scheduling period as the individual nurse's schedule; whereas the overall timetable for all nurses (all schedules) is denoted as the nurse roster.

NRP is to assign each available nurse in a specific category to an individual schedule, i.e. a sequence of day-on and day-off duties. On each day-on, the nurse can be assigned to a particular shift (e.g., early, day, evening or night shift). The problem data such as the number of personnel in a ward, the number of personnel in each skill category, the demand of each category of nurses and the definition of shift types, etc. are determined at the earlier stage of staff planning, which is the first stage of the overall nurse workforce management .

There are two general types of nurse rostering cyclical and non cyclical scheduling. Each one has its advantages and disadvantages and is suitable for different situations. In cyclical scheduling, a single schedule for a fixed planning period is created, and assigned to all employees. The scheduling restarts once the end of the planning period is reached. Cyclical scheduling has a number of advantages. As everyone has the same schedule, nurses cannot feel whether their schedule is worse than that of anyone else. Secondly, once a good cyclical scheduling is produced, it can be reused until the scheduling requirements change. Cyclical scheduling does have disadvantages too. It is more challenging when the covering requirements are different from day to day, or week to week. Non cyclical scheduling, as the name suggests, is opposite to cyclical scheduling. Each nurse has a schedule which satisfies their personal preferences and requests. So it is more flexible than cyclical scheduling. However, it is generally much more difficult to solve. In this thesis, the NRPs investigated are all non cyclical.

The constraints in NRPs can vary from one hospital to another while the objectives can also vary. These have resulted in a whole range of NRP models and, correspondingly, a wide range of solution approaches that have been developed for these models. In the following

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section of this chapter, we give a brief overview of the modelling issues of NRP and then finally Hybrid approaches were discussed.

2. Formulating the nurse rostering problem

2.1 Decision variables and domains

The nurse rostering is commonly described by a nurse-day view which is a direct depiction of two-dimensional duty rosters. Accordingly, the decision variables can be defined for each nurse on each day as NS_{ij}, where i index the nurses and j indexes the days within the scheduling period. The domains of the variables consist of day-on and day-off duties. This type of decision variable and domain are widely used in CP.

For example, an off shift with 3 day-on (i.e. early, late, night) shifts can be defined as:

$$\boldsymbol{NS_{ij}} = \begin{cases} \mathbf{1};\ \boldsymbol{if\ nurse\ i\ take\ off\ shift\ on\ day\ j} \\ \mathbf{2};\ \boldsymbol{if\ nurse\ i\ take\ early\ shift\ on\ day\ j} \\ \mathbf{3};\ \boldsymbol{if\ nurse\ i\ take\ late\ shift\ on\ day\ j} \\ \mathbf{4};\ \boldsymbol{if\ nurse\ i\ take\ night\ shift\ on\ day\ j} \end{cases}$$

Nurse	Mon	Tue	Wed	Thu	Fri	Sat	Sun
P	2	2	2	2	1	1	1
Q	4	4	1	1	3	3	3
R	3	3	1	1	2	2	2
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The above Table shows part of a weekly roster where the shifts are allocated to the nurses in a nurse-day view.

The Integer Programming/Mixed Integer Programming are used to applied for 0-1 model, the decision variables are usually customized to be NS_{ijk}, where i, j are the same indexes as that for NS_{ij}, k indexes the possible shifts in a day. In the above example, NS_{ijk} is binary:

$$NS_{ijk} = \begin{cases} 1, if\ nurse\ i\ work\ shift\ k\ on\ day\ j \\ 0, \qquad\qquad\qquad otherwise \end{cases}$$

2.2 Constraints

Constraints vary from different hospitals. Researchers and practitioners tend to define the constraints according to the requirements and situation of their own organizations. This can be seen in a large amount of literature in NRPs. Therefore, it is usually difficult to have a fair comparison among the different solution approaches. In order to provide a test bed for the algorithms we developed in this thesis, we choose to investigate a set of benchmark NRPs. These problems, on the one hand, are from real world thus reflecting the request of hospitals; on the other hand, they are also tested by other researchers, accordingly the different solution approaches can be analyzed and compared fairly. Next, we present the constraints we are going to investigate. These constraints commonly occur in the benchmark NRPs.

- The shift coverage requirements must be fulfilled
- Minimum rest time between shifts
- Maximum number of shift assignments within the scheduling period
- Maximum number of consecutive working days
- Minimum number of consecutive working days
- Maximum number of consecutive non-working days
- Minimum number of consecutive non-working days
- Maximum number of hours worked
- Minimum number of hours worked
- Maximum number of a certain shift type worked (e.g. maximum seven night shifts for the scheduling period)
- Maximum number of a certain shift type worked per week (same as above but for each individual week)
- Valid number of consecutive shifts of the same type
- Free days after night shifts
- Complete weekends (i.e. shifts on both Saturday and Sunday, or no shift over the weekend)
- No night shifts before free weekends
- Identical shift types during the weekend
- Maximum number of consecutive working weekends
- Maximum number of working weekends in four weeks
- Shift type successions (e.g. Is shift type A allowed to follow B in the next day, etc)

- Requested days on or off
- Requested shifts on or off

2.3 Objective functions

Typically, we use standard objective functions in models, such as those in mathematical programming. For example, the objective $\min \sum p_{ij} NS_{ij}$, where p_{ij} is the penalty cost of nurse i working on day j, NS_{ij} are the decision variables, defines the purpose to minimize the total penalty cost for all nurses. In other situations, a penalty function can be used when feasibility cannot be guaranteed. The function is the penalty for violating constraints. This widely happens in over-constrained NRPs.

3. Solution approaches to nurse rostering problems

Several recent surveys of employee scheduling provide a large amount of information about problem models and solution methods. In this section we review the key methods investigated in the thesis (i.e. Constraint programming (CP), Integer Programming (IP), and hybrid approaches) that have been used to solve NRPs of varying complexity. Some of the papers reviewed about Staff scheduling and rostering, A Constraint Programming based Column Generation Approach to Nurse Rostering Problems, etc. In order to provide a new contribution to the research community, we have made an effort to review the papers by highlighting the key points about the feature of the problem and that of the solution methods.

3.1 OR approaches

Linear Programming, and Integer Programming, etc

In this section we review the publications which use Linear Programming and Integer Linear Programming methods to tackle the NRPs. These methods are usually used to solve the 0-1 model. The Integer Program problems are usually solved by Branch-and Bound algorithm, column generation, or Branch-and-Price algorithm, etc. The procedures such as branching strategy, bounding procedure, and column generation procedure are critical to the success of the algorithms.

One of the first exact optimisation approaches to NRPs was presented by Warner and Prawda. The problem is formulated as a Integer Quadratic Programming problem. A solution to the problem represents a staffing pattern which specifies the number of nurses to cover the shifts for six wards. The goal is to minimize the nurse shortage costs while satisfying the total nursing personnel capacity, and the integral assignment constraint. The problem is decomposed into linear 0-1 programming master problem and small quadratic programming sub problems. Each feasible solution to the sub problems is a candidate solution for the master problem. Bailey presents an approach which combines the problem of shift planning and the assignment of those shifts to employees while considering some basic work pattern constraints. The objective is to minimize the understaffing subject to a fixed workforce size and overtime restriction. Linear Programming is efficient to identify the optimal shifts and on-off patterns. The shifts are then matched to the patterns heuristically, aiming to minimize the difference in a nurse shift start time over the period. Mason and Smith describe column generation methods to efficiently solve a NRP using linear and Integer Programming techniques. Columns are generated by dynamic programming solving the shortest path problems with respect to the nurse's preferences for different shifts, and consecutive on-off pattern, etc.

Jaumard et al solve a NRP with the objective of reducing salary cost, improving nurse preference satisfaction. They also use column generation techniques where the columns correspond to individual schedules for each nurse. The sub problem is a resource constrained shortest path problem. Bard and Purnomo combine the heuristic and Integer Programming methods to solve a NRP with up to 100 nurses and approximately 13 hard and soft constraints. The objective of the problem is to minimize the costs incurred from employing outside nurses and to maximize the satisfaction of nurses working preferences. High quality individual nurse schedules are created by using a single or double shift swapping heuristic on a base schedule. These columns are then used to form a set covering problems which is solved by Branch-and-Bound. The authors find that, for most of the instances that the algorithm is tested on, the majority of the computational time is spent on generating columns rather than Branch-and-Bound.More recently, Bard and Purnomo propose a nurse rostering model which combines cyclic and preference scheduling in [95]. The problem is solved using Lagrangian relaxation and Branch-and-Price. Maenhout and Vanhoucke present an exact Branch-and-Price algorithm for NRP incorporating different branching strategies.

As from the publications discussed above, column generation is often used in OR approaches to NRPs. This is due to the feature of the NRPs. The columns in NRPs represent the possible work patterns for individual nurses. The columns are generated by OR algorithms, such as shortest path algorithm, dynamic programming. And the heuristics can be integrated into the column generation procedure by modifying other columns via swapping assignments. More sophisticated methods such as Constraint Programming based column generation emerge more recently.

3.2 Constraint Programming

Darmoni et al use CP to solve the scheduling problems in a French hospital. The model is build based on Charme, a Constraint Programming language, consisting of a certain level of constraints to be satisfied. The solution procedure of the CP model consists of three main parts. Constraint propagation is first performed on each variable domain to deduce reduced domain. A search strategy trying to ensure fair scheduling among nurses is applied. The approach is able to produce satisfactory schedules over a planning horizon up to 6 weeks.

Weil et al apply a CP solver: ILOG solver to solve a NRP with only a number of typical constraints such as minimum day off and consecutive shift pattern constraints. The model of the constraints in ILOG solver as a CP. The system can find one feasible solution or all feasible solutions according to the request of the user. With respect to the soft constraints, the system can only provide the solution satisfying hard constraints and indicate if the solution violates soft constraints or not.

Cheng et al present a CP method for solving a week long NRP in a Hong Kong hospital. A redundant modelling idea is described, which involves formulating the same problem in two distinct ways (shift to nurse and nurse to shift assignment). During the search, both formulations are simultaneously updated and fed back into each other. For each soft constraint, a branching decision is posted. One branch is to add the soft constraint to the model; the other branch is without the soft constraint. A final result to the problems is presented by the percentage of the satisfaction of soft constraints.

Metivier et al propose a hybrid approach which emphasizes the application of soft global constraints. The interaction among the global constraints is investigated through the communication among them. The filtering algorithm can be more efficient when the constraints which share a common set of variables are considered together.

Wong and Chun apply CP to solve the NRP with the help of meta-level reasoning and probability-based order heuristic. The meta-level reasoning is executed before the search to generate redundant or implied constraints from the existing constraints. These new constraints can help in further reducing the search space. Probability-based ordering is used as a value ordering heuristic. It approximates the probability of value assuagements occurring in the solution set and thus uses this information to guide the search.

4. Hybrid approaches

Hofe combines the ideas of heuristic local search with CP techniques to create an automated nurse rostering system tested in a German hospital. It models the problem as a Hierarchical Constraint Satisfaction Problem (HCSP) with fuzzy constraint. The constraints are organised into hierarchies of different priorities to reflect their importance. The fuzzy constraint allows a constraint to be partially satisfied and partially violated. The HCSP are solved by heuristics which are used to identify and repair violations.

Li et al. present a hybrid AI approach to a class of over-constrained NRPs. Their approach has two phases. The first phase solves a relaxed version of the problem which only includes hard rules and part of the nurse's requests for shifts. It applies a forward checking algorithm with non-binary constraint propagation, variable ordering, random value ordering and compulsory back-jumping. In the second phase, the adjustments are made by descend local search and tabu search to improve the solution. The experiments show that the approach is able to solve this class of problem well.

Demassey et al. investigate a CP based column generation approach which emphasizes the cost-filtering algorithms of optimisation constraint. A new optimisation constraint- cost regular for a global constraint- regular. The optimisation constraint links a cost to the decision variable assignments. Its filtering algorithm is based on the computation of the shortest and longest paths in a layered directed graph. The approach is applied to an employee timetabling problem where the columns are generated with the help of cost-regular constraint.

5. Conclusion

In this paper, we have discussed a wide range of nurse rostering problem. The main focus of this paper is to investigate how to efficiently integrate Constraint Programming, Operational Research techniques and hybrid approaches methods to solve a combinatorial optimization problem from real world applications, taking the advantages of each well developed component.

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