Concatenated Codes with Two Inner Coding Schemes in Frequency-Hopping Spread Spectrum Multiple-Access Channels

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Abstract—In this paper, we analyze the performance of a concatenated code with two different inner decoding schemes. One is the error-detecting inner coding, and the other is the error-detecting-and-correcting inner coding scheme. We compare the performance of the two decoding schemes for finite and infinite block length cases when the concatenated code is applied to slow frequency-hopping spread-spectrum multiple access (FH/SSMA) communication systems. Our results show that the error-detecting-and-correcting inner decoding scheme outperforms the error-detecting inner decoding scheme for finite block lengths, while there exists a performance trade-off for infinite block length case.

Keywords: concatenated code; inner code; decoding scheme; frequency hopping; multiple access channel

I. INTRODUCTION

Concatenated codes form a class of error-correcting codes that are derived by combining an inner code and an outer code [1]. They were conceived as a solution to the problem of finding a code that has both exponentially decreasing error probability with increasing block length and polynomial-time decoding complexity. Concatenated codes became widely used in space communications in the 1970s and recently adopted in Digital Television Terrestrial Broadcasting (DTTB) [2].

The most natural choice for outer codes is Reed-Solomon (RS) codes in concatenated codes. Because the RS codes, being maximum-distance-separable codes, make highly efficient use of redundancy, and well suited to burst error correction [3]. We will use RS codes as outer codes throughout this work. The inner code we consider in this paper is error detecting or correcting binary block code. The inner code corrects $e_d$ errors and detects $e_d$ errors provided $2e_d + e_a < d_{min}$, where $d_{min}$ is the minimum distance of the inner code. When an error is detected, every symbol of the inner code is erased. There are, however, errors that are not detected nor corrected by the inner code, which results in errors at the output of the inner decoder. The purpose of the outer code is to correct the errors and erasures of the inner code.

In this paper, we address performance comparisons of concatenated coding systems, which have two decoder types for the inner code. One is to detect errors only. The other is to detect and correct errors. We assume that the concatenated coding is applied to a frequency-hopped spread-spectrum (FH/SS) packet radio network in which $I$ users wish to communicate simultaneously over a common channel. We derive the error detection probability and undetected probability for an inner code. We will examine the corresponding overall block error probabilities for both finite and infinite case of outer code lengths.

This paper is organized as follows. Section II introduces the system model and the channel model. The error detecting inner decoder is presented in Section III. The error detecting and correcting inner decoder is provided in Section IV. The numerical results and discussions are shown in Section V. Finally, a conclusion is made in Section VI.

II. SYSTEM AND CHANNEL MODEL

We consider a FH/SS packet radio channel model with $I$ simultaneous users as shown in Figure 1. The input of the $j$th source generates messages, which are independent of other users. There are $I$ separate encoders, one for each source. The $j$th encoder receives only the message from the $j$th source and produces a codeword $(x_{j,1}, x_{j,2},...,x_{j,n})$. The frequency hopping pattern dehopped the received signal. The output is a super channel which is identical for all users [4].

When an inner code is used for detecting errors, a super channel created by the inner code can be modeled by $M$-ary erasures and errors channel as shown in Figure 2. The purpose of the outer code is to correct the errors and erasures of the inner code. From the minimum distance property [3], the $(n, k)$ Reed-Solomon code with bounded distance decoding can correct up to $e = n - k$ erasures or up to $t = (n-k)/2$ errors. More
generally, it can correct any combination of \( l \) erasures and \( m \) errors provided that \( 2m + l \) does not exceed \( n - k \). Thus the probability of overall block (an outer codeword) error, \( P_E \), for the memoryless channel is given by

\[
P_E = \sum_{2m+l=n-k}^{n} \binom{n}{l,e} P_d^{l} P_{ed}^{e} (1 - P_d - P_{ed})^{n-l-e}
\]

(1)

where

\[
\binom{n}{l,e} = \frac{n!}{l! e! (n-l-e)!}.
\]

(2)

We assume that all FH transmitters adjust their timings of frequency changes (synchronous frequency hopping), and transmit one inner symbol during a hop (fast frequency hopping). Thus, the multi-user interference level during a hop will remain constant throughout the hop. We assume that the hopping pattern is essentially random, which makes the interference during a hop independent of that of the other hop intervals.

When \( I \) users transmit their packets simultaneously, it is probable that \( i+1, \ i \in \{0, 1, 2, \ldots, I-1\} \), users occupy a particular frequency slot simultaneously. If a frequency slot is occupied by \( i+1 \) users, the slot can be modeled by a binary symmetric channel (BSC) \( \Delta_i \) with a channel crossover probability \( p_i \), given by

\[
p_i = \frac{2^i - 1}{2^{i+1}}.
\]

(3)

On the other hand, the probability of the channel \( \Delta_i \) being chosen, i.e. the probability of \( i+1 \) users occupying the same frequency slot, \( P_{h,i}(i) \), is given by

\[
P_{h,i}(i) = \binom{I-1}{i} \left( \frac{1}{q} \right)^i \left( \frac{1}{q} - 1 \right)^{I-1-i}, \ \ i \in \{0, 1, \ldots, I-1\}.
\]

(4)

III. ERROR DETECTING INNER DECODER

When the inner code is used for detecting errors only, the receiver will decode symbols incorrectly if and only if a channel-error pattern is the same as a nonzero codeword [5]. For a BSC, the probability that a channel error pattern will match any particular codeword of weight \( i \) in a code of length \( N \) is equal to \( P^i (1-P)^{N-i} \). Thus the probability of an error pattern being detected is given by

\[
P_{ad} = \sum_{i=1}^{N} A_i P^i (1-P)^{N-i}
\]

(5)

where \( A_i \) is the weight distribution of a code. Thus for the component channel model, \( P_{ad} \) is given by

\[
P_{ad} = \sum_{i=1}^{N} A_i p^i (1-p)^{N-i}
\]

(6)

where

\[
p = \sum_{i=0}^{\infty} p_i P_{h,i}(i)
\]

(7)

The probability of a codeword being correctly decoded is then given by

\[
P_e = \left[ \sum_{i=0}^{N} \binom{I-1}{i} \left( \frac{1}{q} \right)^i \left( \frac{1}{q} - 1 \right)^{I-1-i} \right]^N
\]

(8)

and, thus the probability of error detection can be obtained as

\[
P_d = 1 - P_e - P_{ad}.
\]

(9)

IV. ERROR DETECTING AND CORRECTING INNER DECODER

In this section, we consider an inner decoder that detects \( e_d \) errors and corrects \( e_c \) errors with \( 2e_c + e_d < d_{min} \) and \( e_d < e_c \), where \( d_{min} \) is the minimum distance of the inner code. The probability of correctly decoding a codeword is given by

\[
P_c = \sum_{i=1}^{\infty} p^i (1-p)^{N-i}
\]

(10)

where

\[
p = \sum_{i=0}^{\infty} \binom{I-1}{i} \left( \frac{1}{q} \right)^i \left( \frac{1}{q} - 1 \right)^{I-1-i} \]

(11)

The probability of undetected error can be approximately given by

\[
P_{ud} \approx \sum_{i=0}^{N} \binom{N}{i} p^i (1-p)^{N-i} \left( \frac{2^k-1}{2^k} \right)^{\sum_{j=1}^{K} N_j}.
\]

(12)

Thus the probability of an error pattern being detected is given by
\[ P_d = 1 - P_w - P_e. \] (13)

V. NUMERICAL RESULTS AND DISCUSSIONS

Asymptotically, as \( n \) and \( k \) approach infinity while the outer code rate \( r \) is held constant, it can be shown from the central limit theorem [6]

\[
\lim_{n,k \to \infty} P_e = \begin{cases} 
0, & r < 1 - P_d - 2P_w \\
1/2, & r = 1 - P_d - 2P_w \\
1, & r > 1 - P_d - 2P_w 
\end{cases}
\]

That is to say, error-free communication is possible asymptotically if \( r < 1 - P_d - 2P_w \). Then \( 1 - P_d - 2P_w \) should be maximized. In other words, it is desirable we choose the inner code which has minimum \( P_d + 2P_w \) in asymptotic region.

![Figure 3. \( P_d + 2P_w \) vs. \( I \) for two decoding schemes. (\( q = 25, n = 2048, K = 11, N = 16 \)](image)

For performance comparisons of two inner decoder types, we have plotted \( P_d + 2P_w \) versus \( I \) for two decoder types in Figure 3. We choose \( n \) to be 2048 (large enough to simulate infinite block length). The inner code is an extended-Hamming code. Then \( d_{min} \) of the inner code is 4 [7]. It guarantees \( e_d = 3 \) for being used an error-detecting code, and \( e_d = 2 \) for an error detecting-and-correcting code. It can be noticed that there exists a crossover point in Figure 3. We can see that error-detecting-and-correcting inner code has lower \( P_d + 2P_w \) than error-detecting inner code when the channel traffic is low.

In Figure 4 and Figure 5, we have also plotted \( P_e \) versus \( k \) for \( I = 5 \) and 25, respectively, for comparing two decoder types in finite block length cases. Inner code is also an extended-Hamming code (\( d_{min} = 4 \)). It can be seen that the error-detecting-and-correcting decoder for an inner code has lower block error probability than the error-detecting decoder in most part of the regions. This implies that it is advantageous to adopt the error detecting-and-correcting scheme for the inner decoder for finite value of \( n \). But it was not true for asymptotic case (\( n=2048 \) in our example).

![Figure 4. Plot of \( P_e \) vs. \( k \) (\( I = 5, q = 25, n = 16, K = 4, N = 8 \))] (image)

![Figure 5. Plot of \( P_e \) vs. \( k \) (\( I = 25, q = 25, n = 16, K = 4, N = 8 \))] (image)

VI. CONCLUSIONS

We considered the performances of two types of inner decoding schemes; one detects errors only and the other detects and corrects errors. We assumed that one inner code symbol is transmitted during a hop (fast FH). It was found that the error-detecting-and-correcting scheme gives better performance than the error-detecting scheme for finite value of block length \( n \) and there exists a performance crossing point with an asymptotic case (\( n=2048 \) in our numerical example).
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