Review on Synchronization for OFDM Systems

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Abstract: Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation scheme. It is widely used modulation technique because it has high data rate, high spectral efficiency and robustness to multipath fading channel. One of the major drawbacks of OFDM system is synchronization. It is very sensitive to frequency synchronization errors in the form of Carrier Frequency Offset (CFO). The Carrier Frequency Offset (CFO) causes Inter Carrier Interference (ICI) and destroy the orthogonality of the OFDM system. Therefore it is necessary to perform frequency synchronization. In this paper various Carrier Frequency Offset Estimation methods are presented.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Carrier Frequency Offset (CFO), Inter Carrier Interference (ICI).

I. INTRODUCTION

The OFDM is a special case of multicarrier modulation scheme in which high rate data streams are converted into the number of low rate data streams and then uses parallel orthogonal subcarriers to transmit the data simultaneously. The OFDM is more bandwidth efficient system than the simple FDM provided that the orthogonality between the subcarriers is preserved. The main advantage of using OFDM is to increase the robustness against frequency selective fading or narrowband interference. Due to its advantages like high data transmission rate, high bandwidth efficiency, robustness against multipath fading, the OFDM is widely used in many wireless communication systems. One of the major drawbacks of OFDM system is their sensitivity to synchronization errors. The carrier frequency of the received signal may be different from the transmitted carrier frequency. The difference between the received carrier frequency and signal carrier frequency is called Carrier Frequency Offset (CFO). The Carrier Frequency Offset (CFO) may occur due to mainly two reasons: first is due to mismatch between the local oscillators of transmitter and receiver and second is due to Doppler Effect when the transmitter is in motion relative to the receiver. This is called Doppler shift. The process of estimating the Carrier Frequency Offset (CFO) is called carrier frequency synchronization. Frequency synchronization is the basis of the orthogonality between frequencies. In OFDM systems, the whole concept and all the advantages depend on the orthogonality of sub-carriers. If the orthogonality between sub-carriers lost due to Carrier Frequency Offset (CFO) than the performance of OFDM system degrades. This paper presents the effects of Carrier Frequency Offset (CFO) on OFDM systems and various techniques to estimate the Carrier Frequency Offset (CFO) so that we can correct these effects.

II. SYSTEM MODEL

The OFDM system (Transmitter & Receiver) is shown in Fig. 1. The serial data is given as an input to the transmitter. Suppose the serial data input is $X$. The serial data is first converted into the parallel streams. The serial data $x$ is converted into $x_1, x_2, x_3, \ldots, x_k$. The Inverse Fast Fourier Transform (IFFT) is performed on parallel data stream. Then the parallel data stream is converted back to the serial data and guard interval is inserted.

A guard interval is usually inserted between successive OFDM symbols to avoid the Inter Symbol Interference (ISI). If a guard interval with no signal transmission is inserted then the ISI can be eliminated almost completely, but a sudden change of waveform contains higher spectral components, so they result in Inter Carrier Interference (ICI). To avoid the Inter Carrier Interference (ICI), a guard interval with cyclic prefix is generally used. A copy of the last part of the OFDM symbol is attached to its front is called a Cyclic Prefix (CP). Fig. 2 shows OFDM symbol with Cyclic Prefix. In Figure 2 $T_g$ is the guard time interval, $T_u$ is the symbol time interval without cyclic prefix and...
\( T_{\text{sym}} \) is the total time interval of the symbol with cyclic prefix. After inserting the guard interval the Digital to Analog Conversion is performed by Digital-to-Analog Converter (DAC). Finally the analog data is passed through the channel. The signal is received by the receiving antenna. The analog data is converted to digital data by Analog-to-Digital Converter (ADC). Then the guard interval is being removed from the data. After that the serial data is converted to the parallel data streams. The received serial data in the presence of frequency offset is given by,

\[
r_n = e^{j2\pi f_n} \sum_{l=0}^{L-1} S_{n-l}h_l + w_n
\]

(2)

where, \(0 \leq n \leq N - 1\)

Where, \( f_n \) is the frequency offset, \( h_0, h_1, ..., h_{L-1} \) is the impulse response of the channel, and \( w_n \) is Additive White Gaussian Noise (AWGN).

The Fast Fourier Transform (FFT) is performed on the received sequence \( r_n \). The output of FFT is given by

\[
Y_k = \sum_{n=0}^{N-1} r_n e^{-j2\pi nk/N}
\]

(3)

The parallel data stream is then converted to the serial data by using Parallel-to-Serial Converter.

### III. EFFECT OF CARRIER FREQUENCY OFFSET

Frequency offset occurs due to many sources, such as Doppler shift or frequency mismatch between transmitter and receiver oscillators. The first source of error arises when there is relative motion between transmitter and receiver. In this case, the frequency shift is given by,

\[
\Delta f = \frac{v}{c} f_c
\]

(4)

\( \Delta f \) is the frequency shift due to relative motion. For example, with a carrier frequency of \( f_c = 5 \) GHz and a velocity of 100 km/h, the offset value is \( \Delta f = 1.6 k\text{Hz} \), which is relatively insignificant compared to the carrier spacing of 312.5 kHz.

The other source of frequency offset is due to frequency errors in the oscillators. The IEEE 802.11a standard requires the oscillators to have frequency errors within 20 ppm (or \( \pm 20 \times 10^{-9} \)).

For a carrier of 5GHz, this means a maximum frequency error of

\[
|\Delta f_{\text{max}}| = 2 \times 20 \times 10^{-6} \times 5 \times 10^9
\]

Where the factor 2 is for the transmitter and receiver having errors with opposite signs. This error is relatively large compared to the frequency spacing of the carrier.

OFDM systems are more sensitive to frequency errors than Single Carrier Modulation (SCM) systems. In OFDM systems, a frequency offset destroys orthogonality between carriers and introduces Inter Carrier Interference (ICI). This is shown in Fig. 3. The areas, colored with yellow, show the Inter Carrier Interference (ICI). When the centers of adjacent subcarriers are shifted because of the frequency offset, the adjacent subcarriers nulls are also shifted from the center of the other subcarrier. The received signal contains samples from this shifted subcarrier, leading to Inter Carrier Interference (ICI).

<table>
<thead>
<tr>
<th>Table 1: The Effect of CFO on the Received Signal</th>
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<tbody>
<tr>
<td>Time domain signal</td>
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<tr>
<td>--------------------</td>
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<tr>
<td>( y[n] )</td>
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<tr>
<td>Frequency domain signal ( Y[K] )</td>
</tr>
</tbody>
</table>

\[
\text{Figure 3. Inter Carrier Interference (ICI) subject to Carrier Frequency Offset (CFO)}
\]

### IV. CARRIER FREQUENCY OFFSET ESTIMATION METHODS

The destructive effects of the frequency offset can be corrected by estimating the frequency offset itself and applying proper correction. This requires the development of a frequency synchronization algorithm. Due to the increasing importance of OFDM in wireless communication, a number of techniques have been developed.
Typically, three types of algorithms are used for carrier frequency offset estimation: 1. Data Aided 2. Non Data Aided and 3. Blind

**A. DATA AIDED METHOD:**

Data Aided method uses additional preambles or training sequence or pilot sequence for frequency offset estimation [1]. The training sequences can be designed in such a way that they can limit the number of computation at the receiver side and hence this method has a low computational complexity [2].

There have been several methods proposed by several authors.

The most popular method is proposed by Schmidl et al. [3]. The form of the training symbol proposed by Schmidl is \([A\ A]\), where A represents samples of length \(L=N/2\). In this method autocorrelation of a training symbol with two identical parts is used to estimate timing and fractional frequency offset. An additional training symbol is then used along with the first to determine the integer frequency offset. However, the timing metric of Schmidl’s method has a plateau, which causes a large variance in the timing estimate and this leads to degradation of (Bit Error Rate) BER performance.

Morelli et. al. [4] extend the algorithm proposed by Schmidl. This method uses a training symbol composed of more than two identical parts. This method achieves better accuracy but it increases the computational complexity. The estimation range can be made large without the use of a second training symbol, as required by the Schmidl’s method.

Hyoung-Kyu Song et. al. [5] proposed a multistage technique for frequency synchronization. This technique uses a repetitive signal structure inside of a OFDM symbol which is used to enlarge the range and increase the accuracy of the offset estimation.

The above OFDM synchronization methods have one or more of the following drawbacks: have a limited range of operation, have a large estimation variance, lack robust sync detection capability, and requires extra overheads. To overcome these limitations Mimm et. al. [6] proposed a method for OFDM synchronization. In this method a training symbol of identical parts with different signs is used. For example training symbol for \(L\) identical parts is given by: \([-A\ -A\ -A\ -A]\) for \(L=4\).

**B. NON-DATA AIDED METHOD:**

In Non-Data Aided method cyclic prefix is used for synchronization.

It is bandwidth efficient as there is no need for additional preamble. However the accuracy of this type of estimation method is less as Cyclic Prefix (CP) is a part of the PFDM data and is contaminated by Inter Symbol Interference (ISI) [1].

The most common Non Data Aided estimation method uses Maximum Likelihood (ML) algorithm.

Van de Beek et. al. [7] proposed non data aided method of CFO estimation. This method uses the redundant information contained within the cyclic prefix. A log likelihood function for \(\theta, \epsilon\) is

\[
\Lambda (\theta, \epsilon) = \log f(\theta, \epsilon) = |\gamma(\theta)| \cos (2\pi \frac{\epsilon}{M} + \theta) - \rho \phi(\theta)
\]

Where \(\Lambda\) denoted argument of a complex number.

\[
\gamma(m) = \sum_{k=m}^{m+L-1} r(k)r^*(k+N)
\]

\[
\phi(m) = \frac{1}{2} \sum_{k=m}^{m+L-1} |r(k)|^2 + |r(k+N)|^2
\]

And

\[
\rho = \frac{SNR}{SNR + 1}
\]

The maximization of log likelihood function can be performed in two steps:

\[
\max_{(\theta, \epsilon)} \Lambda (\theta, \epsilon) = \max_{\theta} \max_{\epsilon} \Lambda (\theta, \epsilon)
\]

\[
\theta = \max_{\theta} \Lambda (\theta, \hat{\epsilon}_{ML}(\theta))
\]

The maximum with respect to the frequency offset \(\epsilon\) is obtained when the cosine term in (6) equals one. This yields the ML estimation of \(\epsilon\)

\[
\hat{\epsilon}_{ML}(\theta) = -\frac{1}{2\pi} \angle \gamma(\theta) + n
\]

Where \(n\) is an integer.

**C. BLIND ESTIMATION METHOD:**

Blind CFO estimation method uses the statistical properties of the received signal. In this method the receiver doesn’t have any knowledge about the data that the transmitter has been sending and hence this type of method has a high computational complexity. The high computational complexity is the disadvantage of this method. This method doesn’t use a training sequence as in the case of data aided method and also doesn’t use cyclic prefix as in the case of non data aided method.

The various methods for Blind CFO estimation are as follows:

1. MUSIC method,
2. ESPRIT method,
3. Constant modulus based method,
4. Cyclostationary based method,
5. Kurtosis based method,
6. Tri-linear decomposition based method
H. Liu et. al. [8] proposed a high performance/low complexity blind carrier frequency offset estimation algorithm. This algorithm exploits intrinsic structure information of OFDM signals. The algorithm offers the accuracy of a super resolution subspace method, viz. MUSIC, without involving computationally intensive subspace decompositions. This algorithm takes advantage of the known structure of the subspace of OFDM signals and thus offers performance comparable to the subspace-based algorithms with minimum cost.

U. Tureli et. al. [9] proposed a new technique for blind synchronization of OFDM communications over fading channels encountered in digital broadcasting and cellular communications. ESPRIT-like blind estimation algorithm exploits the structure information of OFDM signals with low fixed complexity. The standard ESPRIT algorithm exploits the shift-invariant structure and estimates the CFO through subspace decomposition and generalized eigenvalue calculation and is not inherently limited in range to a portion of the unit circle. In OFDM, the shift-invariant structure that enables ESPRIT manifests itself directly in the received signal.

M. Ghogho et. al. [10] proposed a CFO estimation method for frequency selective fading channels. In this method a case where the transmitted symbols have constant modulus, i.e. PSK constellations are considered. This algorithm outperforms previous blind methods that exploit the fact that practical OFDM systems are not fully loaded. This algorithm is consistent even when the system is fully loaded. The proposed CFO estimator is obtained via a one-dimensional search and achieves a substantial gain in performance.

B. Park et. al. [11] proposed a cyclostationary based blind CFO estimation algorithm. The estimator exploits the second order cyclostationarity of received signals and then uses the symbol timing and carrier frequency offset information appearing in the cyclic correlation. Since it is a blind estimator, no channel impulse response information is required.

Y. Yao et. al. [12] proposed a low complexity blind CFO estimator for OFDM systems relying on a kurtosis-type criterion. The performance of this blind CFO estimator depends on the channel’s frequency selectivity and the input distribution.

The subspace based methods like MUSIC and ESPRIT have high accurate CFO estimation, but their necessity of virtual carriers leads to spectrum inefficiency. X. Zhang et. al. [13] proposed a new blind CFO estimation method. In this method received signal is denoted as a trilinear model, then the trilinear decomposition based CFO estimation algorithm is proposed. Comparing to both ESPRIT method and the cyclostationary (CS) approach, this algorithm has improved CFO estimation performance. Also this algorithm can work in condition of no virtual carrier.

V. PERFORMANCE MEASURES

A. MEAN SQUARE ERROR (MSE)

The Mean Square Error of an estimator measures the average of the errors, which is the difference between the estimator and what is estimated. Thus the Mean Square Error of CFO measures the difference between the CFO and estimated CFO. The MSE is the second moment of the error, and thus incorporates both the variance of the estimator and its bias. For an unbiased estimator, the MSE is the variance of the estimator.

The Error Variance of the CFO is given by [14],

$$\text{Error Variance of CFO} = E \left( | f_e - \hat{f}_e |^2 \right)$$

Where $f_e$ is the carrier frequency offset and $\hat{f}_e$ is the estimated value of carrier frequency offset.

B. SIGNAL-TO-NOISE RATIO (SNR)

It is the ratio of signal power to the noise power. The unit of SNR is dB.

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

Where $S$ = Signal Power and $N$ = Noise Power

VI. CONCLUSION

In this paper one major drawback of Orthogonal Frequency Division Multiplexing (OFDM) is considered that is the sensitivity of frequency synchronization. The effect of Carrier Frequency Offset (CFO) on OFDM is discussed. Also the techniques of CFO estimation which are Data Aided, Non Data Aided and Blind estimation algorithms are discussed.

REFERENCES


