

HHT Based Analysis of Non Stationary Signals and Metal Structures

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Abstract—Hilbert-Huang Transform (HHT) is an innovative data-processing technique for analyzing non stationary and nonlinear signals. The analysis of these signals is to transform the time-domain data to frequency versus time data instead of the amplitude versus frequency. This paper investigates techniques to apply HHT for locating the instantaneous frequency in a signal. This signal processing technique helps in identifying several frequency components which are the indicators of the problems present in the system under test. This can be applied for analyzing aircrafts' body structure, biomedical signals and seismic signals. Monitoring of civil structures such as bridges and buildings is critical for long-term operational cost and safety of aging structures. Applying HHT to these signals obtained from the various sensors placed in the vicinity of event or entity, it is possible to identify the problems.

Keywords —Hilbert-Huang Transform, Intrinsic Mode Functions, Hilbert Transform, Empirical Mode Decomposition

I. INTRODUCTION

Hilbert-Huang Transform (HHT) [1] is a non-stationary signal processing method presented by Professor Norden E. Huang from the United States in 1998, and improved in 1999. The main innovations of this method are intrinsic mode function (IMF) and empirical mode decomposition (EMD). Through EMD, the signal is decomposed into several IMFs (generally for a limited number), to each of which Hilbert Transform is applied to get meaningful instantaneous frequency; the frequency gives the exact expression of time-frequency spectrum in non-stationary signals. Signal processing techniques employing HHT finds many applications. One such application is detection of human activity behind barriers such as walls and debris when looking for earthquake survivors. The preferred sensors are radars since they have the ability to penetrate deep through dielectric barriers. These sensors are used to recognize signs of life by recognizing micro-Doppler signatures of human activity, such as arm swinging, breathing etc., Such movements induce different types of Doppler spectra depending on the manner in which the limbs and other body parts move, which can be analyzed by several well-known time-frequency approaches, including the empirical mode decomposition and Hilbert-Huang Transform. HHT applied to any nonlinear signal or non-stationary signal obtained from the sensors will yield a good time-frequency plot so as to detect the human activity.

As majority of real-world signals are non-stationary, Fourier analysis provides unsatisfying results since the frequency content changes with time. Determination of the frequency content of such signals dictates to perform an analysis across a span of time (basis function), and then move to another time position. The

major drawback of most Time Frequency transforms is that the basisfunctions are fixed, and do not necessarily match the varying nature of signals. The net effect of these operations is to transform the time-domain data to frequency versus time data instead of amplitude versus frequency variation that the FFT provides.

The Hilbert transform is a well-known method for computing the instantaneous frequency of any signal, under the assumption that only one frequency will be present at any time. Such method cannot directly be applied to a complex signal, containing several frequencies at any given time. EMD technique proposes to decompose a multi-modal signal into a sum of mono-contribution functions called Intrinsic Mode Functions (IMFs) [3]. The EMD is an iterative method that picks out the highest frequency components that remains in the signal at each iteration.

The frequency analysis based on Hilbert-Huang Transform has basically three steps of processing. To begin with the signal is decomposed into a number of IMFs using EMD. In practice, it can be demonstrated that this decomposition process is complete, adaptive and local. The second step applies the Hilbert transform to each IMF to compute the instantaneous frequency at each time. The third and the final step computes the energy content of the considered frequency band at any given time.

II. HILBERT HUANG TRANSFORM

The empirical mode decomposition is the core of HHT which when combined with the Hilbert Transform [1] completes the HHT process. This has been developed from the simple assumption that every signal consists of different simple intrinsic independent modes of oscillations. Each linear or non-linear mode will have the same number of

extrema and zero crossings and only one extremum between successive zero-crossings. In this way, each signal could be decomposed into a number of intrinsic mode functions (IMFs). In contrast to the Fourier spectral analysis in which a series of sine and cosine functions having varying amplitudes are used to represent each constituent frequency components in the signal, the HHT technique is based on the instantaneous frequency calculation that results from the Hilbert transform of the signal. The Hilbert transform $H[x(t)]$ for any signal $x(t)$ is defined as

$$H[x(t)] = y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \left(\frac{x(u)}{t-u} \right) du \quad (1)$$

where P indicates the principal value of the singular integral. The Hilbert Transform can also be interpreted as a natural $\pi/2$ phase shifter, which consists of passing $x(t)$ through a system that leaves the magnitude unchanged, but changes the phase of all frequency components by $\pi/2$. With this definition, $y(t)$ forms the complex conjugate of $x(t)$ and the analytical signal $z(t)$ is defined as

$$z(t) = x(t) + i y(t) \quad (2)$$

As an example, the time and frequency spectrum of a signal, is as shown in Figure 1. From the frequency spectrum, it is

noticed that two frequency components 4Hz and 15 Hz exists but its presence is not indicated in the time domain.

Now applying STFT with a Gaussian window for the time-frequency localization, its spectrogram plot is shown in this figure for different window lengths of 32, 64, 128 and 256. It is observed that the resolution in the frequency domain increases as the width of the window increases and the time resolution decreases.

Equation (2) can be rewritten in a polar coordinate system as

$$Z(t) = a(t) e^{i\theta(t)} \quad (3)$$

Where $a(t) = \sqrt{x(t)^2 + y(t)^2}$ is the amplitude and

$$\theta(t) = \arctan \left(\frac{y(t)}{x(t)} \right) \text{ is the phase} \quad (4)$$

Rewriting in the polar co-ordinate form

$$x(t) = R(z(t)) = R(a(t)e^{j \int w(t) dt}) \quad (5)$$

The Hilbert and the instantaneous frequency is calculated from the above equations

The signal decomposition is based on the following assumptions which need to be satisfied by the EMD process [14] using the algorithm shown in Figure 2.

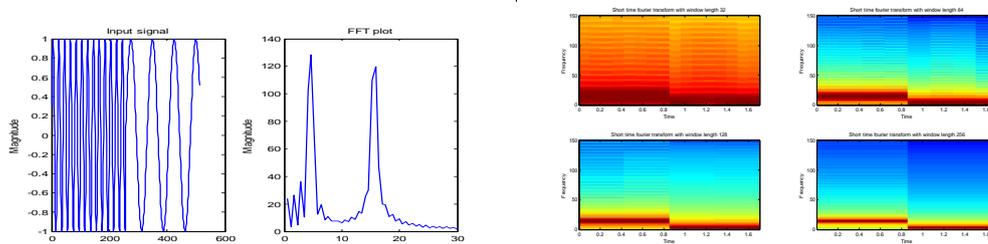


Figure 1. Time and Frequency domain representation and Spectrogram for different window lengths

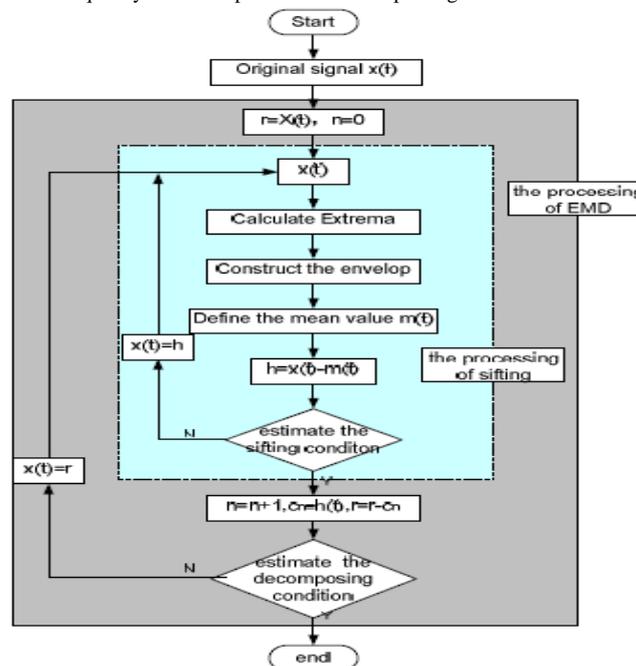


Figure 2. Flow chart of the EMD process

III. EMD PROCEDURE

The EMD process is also known as the shifting process. In general, most of the data are not naturally IMFs and the Hilbert transform cannot provide the full description of the frequency content if the data involves more than one oscillatory mode at a given time. Hence, there is a need to find a way to decompose the data into a set of independent IMF components. Huang introduced a method to decompose a complicated data into IMF components with meaningful instantaneous frequencies. This new method is intuitive, direct, a posterior and adaptive. The decomposition is based on three assumptions:

- a) The signal has at least two extrema, one maximum and one minimum.
- b) The characteristic time scale is defined by the time lapse between the extrema.
- c) If the data were totally devoid of extrema but contained only inflection points, then it can be differentiated once or more times to reveal the extrema.

To find the IMFs of a signal the sifting process consists of several steps and are described using an arbitrary signal denoted $x(t)$.

- (1) Find the positions and amplitudes of all local maxima and minima of the input signal[2].
- (2) Create the upper envelope by spline interpolation of the local maxima and the lower envelope by spline interpolation of the local minima, denoted by $e_{\max}(t)$ and $e_{\min}(t)$.
- (3) At a time instant t , calculate the mean of the upper envelope and the lower envelope m_1 .

$$m_1 = (e_{\max}(t) + e_{\min}(t)) / 2 \quad (6)$$

- (4) Subtract the envelope mean signal from the input signal.

$$h_1(t) = x(t) - m_1 \quad (7)$$

This is one iteration of the sifting process. The next step is to check if the signal $h_1(t)$ is an IMF or not. In the original work of Huang, the sifting process stops when the difference between two consecutive siftings is smaller than a selected threshold standard deviation (SD) [7], defined by

$$SD = \sum_{t=0}^T \left[\frac{|(h_{1(k-1)}(t) - h_{1k}(t))|^2}{h_{1(k-1)}^2(t)} \right] \quad (8)$$

- (5) If $h_1(t)$ is not an IMF, iterate by repeating the process from step (1) with the resulting signal from step (4). Therefore in the second sifting process, $h_1(t)$ is treated as the data resulting in,

$$h_{11} = h_1 - m_{11} \quad (9)$$

Repeat these sifting procedure k times, until h_{1k} is an IMF, that is

$$h_{1k} = h_{1(k-1)} - m_{1k} \quad (10)$$

When the stop criterion is met, the IMF is defined as

$$c_1 = h_{1k} \quad (11)$$

After the IMF c_1 is found, define the residue r_1 as the difference of this IMF and the input signal

$$r_1 = x(t) - c_1 \quad (12)$$

- (6) The next IMF is found beginning from step(1), with the residue as the input signal.

Steps (1) to (6) can be repeated for all the subsequent residues r_j and the result is

$$r_1 - c_2 = r_2, r_2 - c_3 = r_3, \dots, r_{n-1} - c_n = r_n \quad (13)$$

The EMD is completed when the residue, ideally, does not contain any extrema points. This means that it is either a constant or a monotonic function. The signal can be expressed as the sum of IMFs and the last residue

$$x(t) = \sum_{i=1}^n c_i + r_n \quad (14)$$

The extracted IMFs are symmetric and have a unique local frequency and different IMFs do not exhibit the same frequency at the same time.

IV. HHT ANALYSIS PROCEDURE

HHT generation is done based on equations (1) through (4) and (14), with equation (5) modified as

$$x(t) = R \left(\sum_{i=1}^n a_i(t) e^{j \int w_i(t) dt} \right) \quad (15)$$

in which $a_i(t) = \sqrt{[c_i(t)]^2 + H[c_i(t)]^2}$ and $w_i(t) = \frac{d}{dt} \left(\tan^{-1} \frac{H[c_i(t)]}{c_i(t)} \right)$.

The term r_n in (14) is not included in (16) as it is a monotonic function [5] and does not indicate the frequency content of the signal.

Comparing (15) with the Fourier-based representation of a signal $x(t)$ given by

$$x(t) = R \left(\sum_{i=1}^{\infty} a_i e^{j \Omega_i t} \right) \quad (16)$$

where both A_i and Ω_i are constant, it becomes evident that the EMD process enables flexible representation of a dynamic signal by revealing its time-dependent amplitude and the characteristic frequency components at various time instances. The signal is thus represented by a time-frequency

distribution. The underlying HHT of the signal is mathematically defined as,

$$HHT(t, w) = \sum_{i=1}^n HHT_i(t, w) = \sum_{i=1}^n a_i(t, w_i) \quad (17)$$

where $HHT_i(t, w)$ represents the time-frequency distribution obtained from the i^{th} IMF of the signal.

V. EXPERIMENTAL VERIFICATION

HHT analysis is done for two case studies. The first study examines the rationale of HHT for analyzing the gear fault analysis in vehicles. The second is metal plate data analysis which studies the damage which occurs on the plate when a bullet is shot from an air gun from a small distance. The basic concept of HHT is first presented where the empirical mode decomposition must be applied on the signal using a sifting process to obtain intrinsic mode functions before the Hilbert spectral analysis can be meaningfully performed. The wavelet transform was also compared with HHT in the previous work done by us [14] which gave results similar to STFT. HHT is studied for two different cases.

Case Study1: Gear fault analysis in vehicles

To study the performance of HHT for the gear fault analysis, the test signal is generated considering the meshing frequencies at 300Hz and 500Hz, with the Gaussian random noise added. The presence of two impulses at 0.05 and 0.1833 seconds in the system are added to represent the fault as shown in Figure 3. The two frequencies that are present for the entire duration, with the impulse at the specified time are reflected in the HHT plot as shown in Figure 4.

Case Study2: Metal plate data analysis

This case study illustrates the feasibility of the HHT as a signal processing tool for locating an anomaly, in the form of a crack, delamination, stiffness loss or boundary in metal plate, based on physically acquired propagating wave signals. This can be extended to study the external body surface of aircrafts based on simple wave propagation concepts using flight times and speed and the corresponding frequency changes.

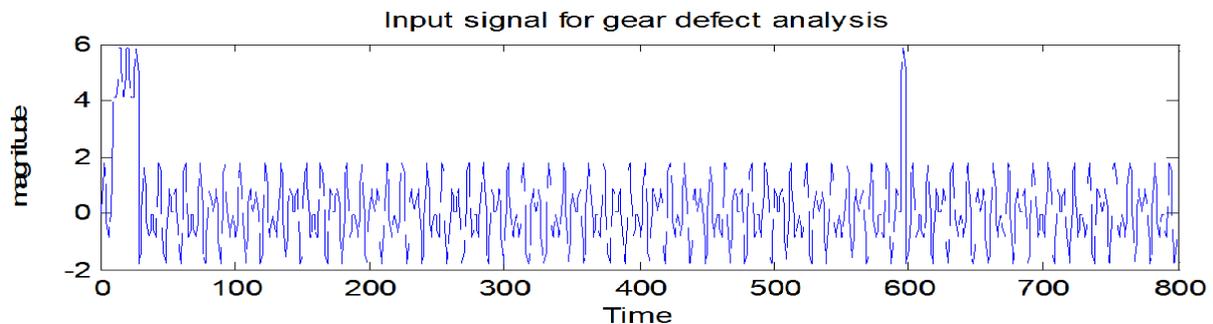


Figure 3. Input signal $s(t)$ with impulse for gear defect analysis

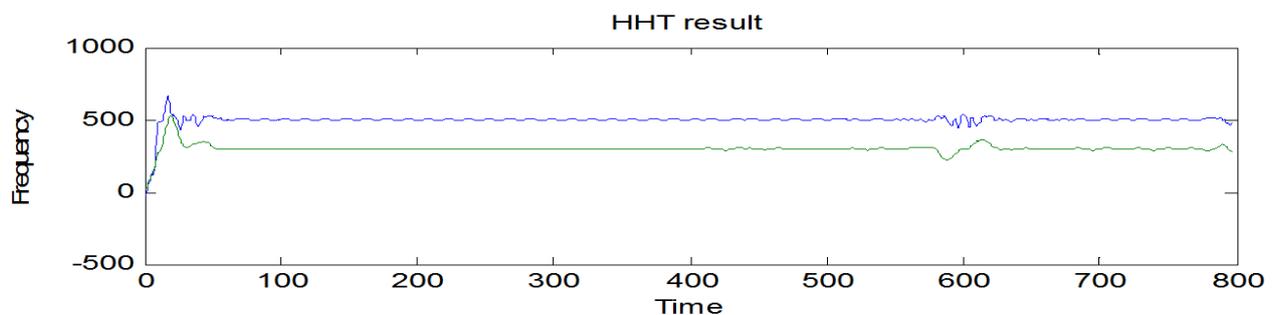


Figure 4. HHT plot for the signal $s(t)$ with an impulse

The data is obtained from a metal plate fitted in the front of an aircraft. For experimental purpose, the plate is exposed to air gun shots at different speeds and the data corresponding to the damage done is collected. With respect to the speed, the damage on the plate in percentage is measured. For three values of 5%, 20% and 30%, the HHT results are plotted to measure the extent of damage by

analyzing the Time-Frequency plot. Figures 5...8 provides the spectrogram, EMD plots and HHT plots for undamaged and damaged plate. It is seen that as the extent of damage increases, the HHT plot shows the increased number of frequency components. By appropriate calibration the extent of the damage is quantified.

Undamaged plate:

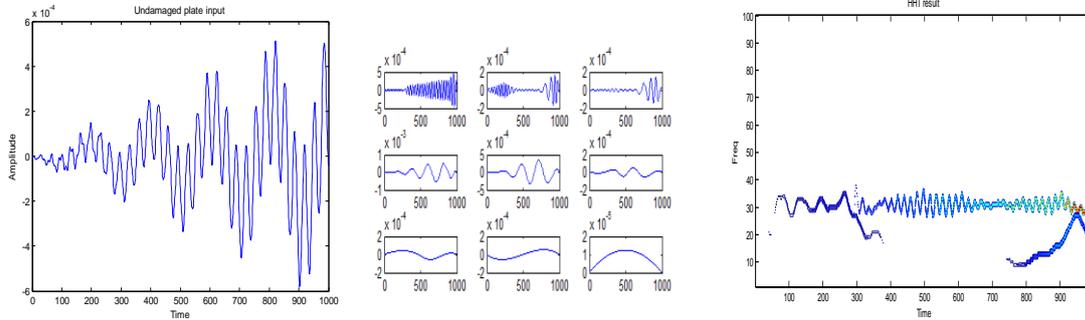


Figure5. Input signal, EMD plot and HHT plot for the undamaged plate

Damaged plate(5%,20%,30%):

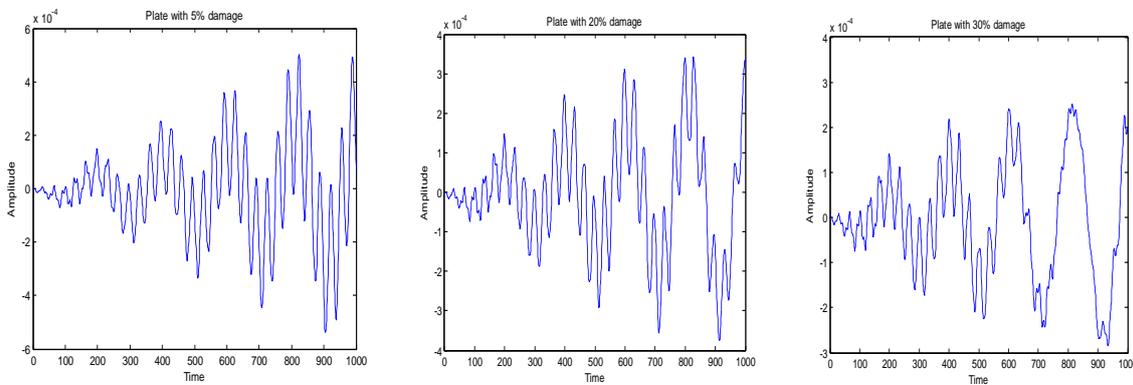


Figure 6. Input with 5%, 20% and 30% damage

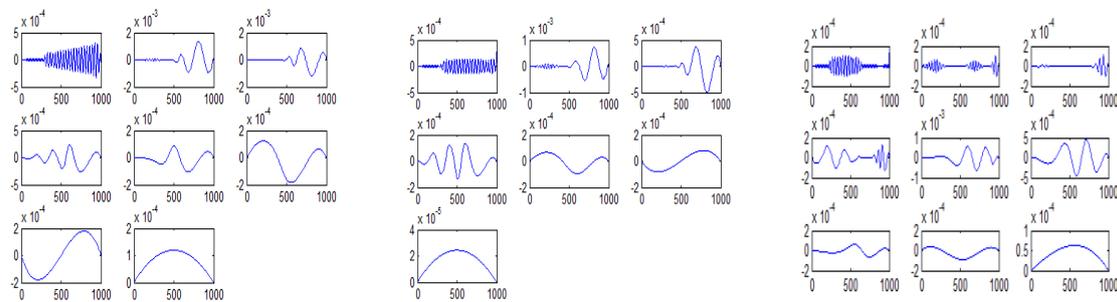


Figure 7. EMD plot for 5%, 20% and 30% damage

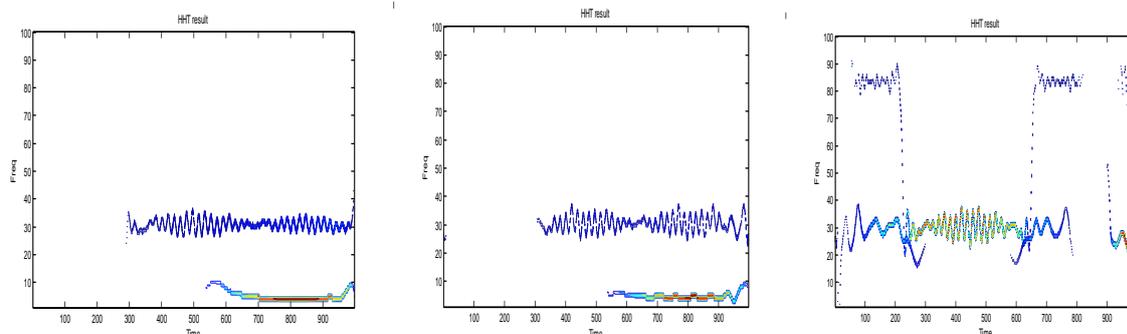


Figure 8. HHT plot for 5%, 20% and 30% damage

VI. CONCLUSION

HHT is a useful tool to get time-frequency representation. Using its multi-resolution properties, concatenated sine signals can be decomposed to get the time-frequency characteristics of signals. It can also be applied for health monitoring of civil structures such as bridges, biomedical signal such as EEG, etc., In addition, using EMD multi-scale filtering features, we can effectively remove the noise, and retain sufficient characteristics of the signal.

The future work will include sensor networks for autonomous structural health monitoring that addresses the synergetic issues of integrating a sensor network with a vibration-based SHM method.

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