Estimation of Mean Time between Failures in Two Unit Parallel Repairable System

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***Abstract*--**Mean time between failures is a method for estimating the reliability parameters of any repairable system. MTBF is also helpful in performing decision analysis in parallel and series systems and subsystems. The MTBF is the reciprocal of the failure rate when each component which fails is replaced immediately with another having the identical failure rate. There are situations when the assumption of a constant failure rate is not realistic and in many of these situations one assumes instead that the failure rate function increases or decreases smoothly with time i.e. there are no discontinuity or turning points. In this paper, we have tried to estimate MTBF taking real failure rate in case of two unit parallel repairable system and study its consistency with either the initial or the last stage of the failure rate curve.

Keywords: *Mean time between failures, reliability, failure rate curve, repairable system.*

Subject Classification: *53A04, 53A05*

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1. Introduction

In many real life situations, more than one failed components can be repaired together. As a result of this, the life of any system can be increased considerably and therefore there is a chance in improvement in mean time between failures. Reliability measures for a two unit parallel system with parallel repair facility was discussed by Rau [2], taking the assumption that the failure and repair times of the components are independently and identically distributed. Several authors before this such as Venkatakrishnan et al[3], Klien and Moeschberger et al[5], Osaki et al [4]and Dharmadhikari et al[6] have considered standby, parallel and series systems with parallel facility in which the component failure times and repair times are not necessarily independent. Recently, Pathak et al[8] have done comparative analysis of reliability parameters taking two different types of distributions. In this paper, we have considered a multivariate exponential distribution of Marshall and Olkin [1] for the failure and repair times and discussed various reliability measures. If we make the exponential assumption about the distribution of failure times, some very useful results can be derived concerning MTBF - the mean time between failures for series and parallel systems. For doing this, we shall have to first obtain a relation expressing the reliability of a component in terms of its service times T= t.

Making use of the fact that

$R(t) = 1 - F(t) = 1 - \int_0^t f(x)\,dx$, we obtain that

$R(t) = 1 - \int_0^t \alpha e^{-\alpha x}\,dx = e^{-\alpha t}$ for the reliability function of the exponential model.

Thus, if a component has a failure rate (here α) of 0.05 per thousand hours, the probability that it will survive at least 10,000 hours of operation (taking t=10) is $e^{-(0.05)10} = 0.607$

Now, suppose that a system consists of n components connected in series, and that these components have the respective failures rates as α_1, α_2,………and α_n. Then the reliability function of the series system can be expressed as

$$R(t) = \prod_{i=1}^{n} e^{-\alpha_i t} = e^{-t\sum_{i=1}^{n}\alpha_i}$$

and the expression for MTBF can be written as $$\mu_s = \frac{1}{\frac{1}{\mu_1}+\frac{1}{\mu_2}+.........+\frac{1}{\mu_n}}$$

Where μ_i is the MTBF's of its components.

For parallel systems, results are however interesting. If a system consists of n components in parallel configuration, having the respective failure rates α_1, α_2,…………., α_n, the system , then the expression for Mean time between failures is given by

$$\mu_p = \frac{1}{\alpha}[1+\frac{1}{2}+.........+\frac{1}{n}]$$

Where α is the identical failure rate.

2. Literature Review

MTBF has been used as a tool for making decisions around 50 years now. Various methods and procedures of estimating MTBF have been developed during these years. As the demand for best cost producer gets higher and higher , more research on finding methods of estimating MTBF have come to light as it is considered most effective tool in this field. The main thrust has been in improving the reliability of system after the improvement in the methods of estimating the MTBF.

3. Problem Definition

Consider a two component system with parallel repair facility in which component failure and repair times follow exponential distribution. Whenever, a component fails it is immediately repaired. If the repair facility is not immediately available it waits for repair. Components are repaired on first come first serve (FCFS) basis.

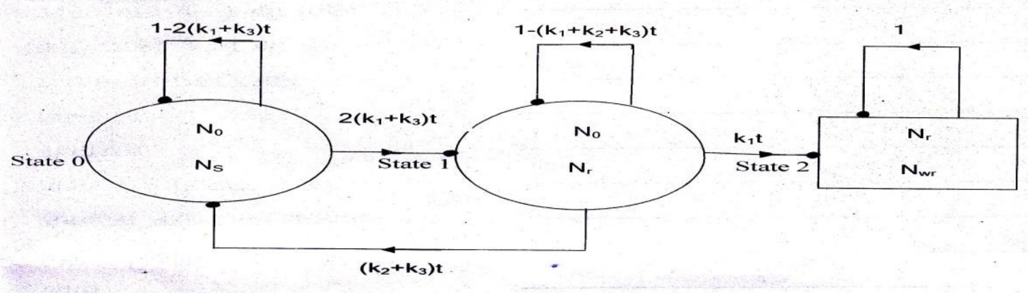

Figure 1 State Transition Diagram

N_o : Unit in operation N_s : Unit in Standby mode

N_r : Unit under repair N_{wr}: Unit waiting for repair

Let X (t) denotes the number of failed components at time t. We define

$$p_i(t) = P\{X(t) = i\} \forall i \in S$$

Where S is the state space. Throughout the chapter, we denote that X (0) =0.

Let T_1, T_2 denote the failure times of components and R_1, R_2 denote their repair times. We assume that the components are identical in nature. We consider the following multivariate exponential distribution of (T_1, T_2, R_1, and R_2) with the survival function:

$$\overline{F}(t_1, t_2) = e^{-k_1 t_1 - k_2 t_2 - k_3 \max(t_1, t_2)}$$

, where t_1 , $t_2 > 0$; k_1, $k_2 > 0$; $k_3 \geq 0$

From the state transition diagram given above, we form the following differential equations for $p_0(t)$ and $p_1(t)$..

$$p_0^{'}(t) = -2(k_1 + k_3)\, p_0(t) + (k_2 + k_3)\, p_1(t)$$

$$p_1^{'}(t) = 2(k_1 + k_3)\, p_0(t) - (k_1 + k_2 + k_3)\, p_1(t)$$

Taking laplace transforms on both the side and noting that $L_i(s)=L\{p_i(t)\}$, the Laplace transform of $p_i(t)$. We get the following system of equations.

$$(2k_1 + 2k_3 + s)\, L_0(s) - (k_2 + k_3)\, L_1(s) = 1$$

$$2(k_1 + k_3)\, L_0(s) - (k_1 + k_2 + k_3 + s)\, L_1(s) = 0$$

Applying Cramer's rule for solving these equations, we get

$$L_0(s) = \frac{(k_1 + k_2 + k_3 + s)}{s^2 + (3k_1 + k_2 + 3k_3)s + 2k_1^2 + 2k_1 k_3} \quad and \quad L_1(s) = \frac{2(k_1 + k_3)}{s^2 + (3k_1 + k_2 + 3k_3)s + 2k_1^2 + 2k_1 k_3}$$

If s_1 and s_2 denote the roots of the equation in the denominator of the above expression, then

$$s_1 = \frac{-(3k_1 + k_2 + 3k_3) + \sqrt{(3k_1 + k_2 + 3k_3)^2 - 8k_1(k_1 + k_3)}}{2}$$

and $$s_2 = \frac{-(3k_1 + k_2 + 3k_3) - \sqrt{(3k_1 + k_2 + 3k_3)^2 - 8k_1(k_1 + k_3)}}{2}$$

Obviously, s_1, $s_2 < 0$. Above expressions for $L_0(s)$ and $L_1(s)$ can also be written as

$$L_0(s) = \frac{(k_1 + k_2 + k_3 + s)}{(s - s_1)(s - s_2)} \quad and \quad L_1(s) = \frac{2(k_1 + k_3)}{(s - s_1)(s - s_2)}$$

Now Inverse Laplace –Transform on above two expressions yield the following results for $p_0(t)$ and $p_1(t)$:

$$p_0(t) = \frac{(k_1 + k_2 + k_3 + s)e^{s_1 t} - (k_1 + k_2 + k_3 + s_2)e^{s_2 t}}{s_1 - s_2} \quad and \quad p_1(t) = \frac{2(k_1 + k_3)(e^{s_1 t} - e^{s_2 t})}{s_1 - s_2}$$

Hence the system reliability is given by:

$$R(t)=\frac{s_1e^{s_2t}-s_2e^{s_1t}}{s_1-s_2}$$

Then $MTBF=\int_0^\infty R(t)\,dt=\int\frac{s_1e^{s_2t}-s_2e^{s_1t}}{s_1-s_2}dt=-\frac{s_1+s_2}{s_1s_2}=\frac{3k_1+k_2+3k_3}{2k_1^2+2k_1k_3}$

When there is no repair facility, then

$$MTBF=\int_0^\infty P\{\max(T_1,T_2)>t\}\,dt=\int_0^\infty [1-P\{\max(T_1,T_2)\le t\}\,dt$$

$$=\int_0^\infty 2e^{-(k_1+k_3)t}\,dt-\int_0^\infty e^{-2(k_1+k_3)t}\,dt=\frac{2}{k_1+k_3}-\frac{1}{2(k_1+k_3)}=\frac{3}{2(k_1+k_3)}$$

Thus gain in average life of the system due to repair facility is given by $\frac{k_1+3k_3}{2k_1^2+2k_1k_3}$

4. Availability and Mean Down Times Analysis

In this section, we obtain the availability measures such as steady-state availability, point wise availability and the mean down time of the system. As before, we consider the following differential equations:

$$p_0^{'}(t)=-2(k_1+k_3)\,p_0(t)+(k_2+k_3)\,p_1(t)$$

$$p_1^{'}(t)=2(k_1+k_3)\,p_0(t)-(k_1+k_2+k_3)\,p_1(t)$$

$$p_2^{'}(t)=k_1p_1(t)-(k_2+2k_3)\,p_2(t)$$

Taking Laplace's transform on both the sides of above equations and using Cramer's rule, we get:

$$L_0(s)=\frac{s^2+(k_1+2k_2+k_3)s+k_2(k_1+k_3)}{s^3+(k_1+2k_2+5k_3)s^2+\{(k_1+k_2+3k_3)+k_1(k_2+k_3)\}s}$$

$$L_1(s)=\frac{2(k_1+k_2)s+(k_2+2k_3)(k_1+3k_3)}{s^3+(k_1+2k_2+5k_3)s^2+\{(k_1+k_2+3k_3)+k_1(k_2+k_3)\}s}$$

and $$L_2(s)=\frac{2k_1(k_2+k_3)}{s^3+(k_1+2k_2+5k_3)s^2+\{(k_1+k_2+3k_3)+k_1(k_2+k_3)\}s}$$

Let s_1 and s_2 be the roots of the equation $s^2+(k_1+2k_2+5k_3)s\ \ +\{(k_1+k_2+3k_3)+k_1(k_2+k_3)$

The expression for $L_0(s)$ can be written as $L_0(s)=\frac{s^2+(k_1+2k_2+k_3)s+k_2(k_1+k_3)}{s(s-s_1)(s-s_2)}$

Resolving into partial fraction the above equation and solving it and taking the inverse Laplace-transform of the expression so obtained, we get the following expression for $p_0(t)$, $p_1(t)$ and $p_2(t)$.

$$p_0(t) = [\frac{s_2 e^{s_1 t}\{s_1(s_1 + k_1 + 2k_2 + k_3) + k_2(k_1 + k_3)\}}{(s_1 - s_2)} + \frac{s_1 e^{s_2 t}\{s_2(s_2 + k_1 + 2k_2 + k_3) + k_2(k_1 + k_3)\}}{(s_2 - s_1)}$$
$$+(k_2 + k_3)(k_2 + 2k_3)]\frac{1}{s_1 s_2}$$

$$p_1(t) = [\frac{2(k_1 + k_2)[s_2 e^{s_1 t}(s_1 + k_2 + 2k_3)]}{(s_1 - s_2)} + \frac{s_1 e^{s_2 t}[(s_2 + k_2 + 2k_3) + (k_2 + 2k_3)]}{}]\frac{1}{s_1 s_2}$$

$$p_2(t) = 2k_1(k_2 + k_3)[\frac{s_2 e^{s_1 t}}{(s_1 - s_2)} + \frac{s_1 e^{s_2 t}}{s_2 - s_1} + 1]\frac{1}{s_1 s_2}$$

We now get the point wise availability of the system A(t) by $p_0(t)+p_1(t)$.

Thus, $$A(t) = [\frac{(k_2 + 2k_3)(k_1 + 2k_2 + k_3)}{s_1 s_2} - \frac{2k_1(k_2 + k_3)}{s_1 s_2(s_1 - s_2)}](s_2 e^{s_1 t} - s_1 e^{s_2 t})$$

And the steady-state availability of the system A_∞ is obtained by

$$A_\infty = \lim_{t \to \infty} A(t) = \frac{(k_2 + 2k_3)(k_1 + 2k_2 + k_3)}{2k_1 k_2 + k_3(2k_2 + k_3)}$$

And the system mean down time MDT is given by $\frac{MTBF(1 - A_\infty)}{A_\infty} = \frac{k_1 + 2k_2 + k_3}{(k_2 + 2k_3)(k_1 + 2k_2 + k_3)}$

5. Results and Conclusion

The random variables T_1, T_2 and R are independent if and only if $k_3=0$. In our case the above expression clearly depicts that timely repair and proper maintenance of the system is one of the main pulling factors for any Industry. It affects the organization in many ways in terms of annual production, customer satisfaction and company's image. Variation of failure rate versus time is depicted by the Bathtub diagram given below. The useful life time of the figure indicates that a product can be of high MTBF but can have low service life. Flat part of the curve indicates useful life period of the product.

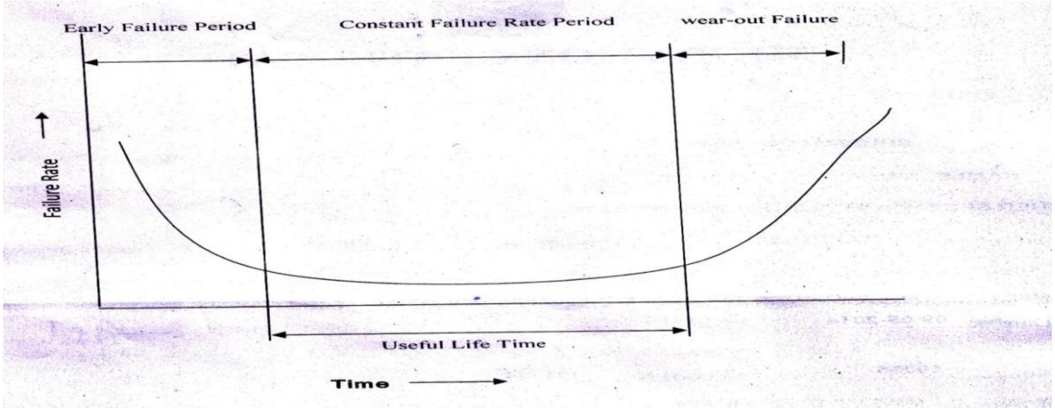

Figure 2 Bathtub Curve illustrating consistent rate of Failure

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